

*Utsav Bali*  
*Calculus Review*  
*1991- 1993*

Orbital Notation ( $\infty$ )

Function Notation  $f(x) = y$

Mapping Notation  $n \mapsto y$

$$x_2 + y_2 = 1$$

$$\text{Eq of a circle} = x^2 + y^2 = r^2$$

$$\left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right) = m$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \frac{\Delta y}{\Delta x}, \text{Under } y, \text{ Under } x$$

$$A(x_2 - x_1)^2 + (y_2 - y_1)^2 = \text{Distance} \rightarrow \text{V.V.V. IMP}$$

$Ax + By + C = 0 \rightarrow$  a line

$y = mx + c \rightarrow$  slope intercept

$y_2 - y_1 = m(x_2 - x_1) \rightarrow$  point slope

Pick point

slope intercept form  $\rightarrow$  numerator moves up or down & den. moves left or right

~~X-Y intercept~~

$\text{X-Y intercept form. solve eq for } x \text{ or } y \text{ \& look at the last value.}$

Equal slope gives  $\parallel$  lines & -ve reciprocals give  $\perp$  lines. **IMP**

Division or multiplication by a  $(-ve \neq 1)$  changes the INEQUALITY sign.

$(-3, 4), (2, 8), (9, 6), (-3, 5)$

$D = \{-3, 2, 9\} \rightarrow$  do not repeat any.

$R = \{4, 8, 6, 5\}$

Relation - a set of ordered pairs where the 1st coordinate forms the domain & the 2nd coordinate is the range.

function  $\rightarrow$  a correspondence that assigns to a # in a certain set, called domain, once & only one member in the second set, called range. **IMP**

$x$  is the independent variable &  $y$  is the dependent value.

$x$  should not be repeated in a set, for the relation to be a FUNCTION.

Vertical line test for functions.

The domain for the eq  $y = mx + b$  will always be all reals i.e.  $(-\infty, \infty)$

$$\Leftrightarrow D = (-\infty, \infty)$$

$$R = \mathbb{R}$$

$$y = \frac{4}{x+2} \quad D = \{n \in \mathbb{R} \mid n \neq -2\}$$

$$D = (-\infty, -2) \cup (-2, \infty) \quad (-2 \text{ is not included})$$

$$y = \sqrt{x+3} \quad \text{cannot have -ve in the radical}$$

$$\therefore D = \{n \in \mathbb{R} \mid n \geq -3\}$$

$$D = [-3, \infty)$$

$$f(x) = x^2 - 3$$

$$f(22) = 22^2 - 3 = 484 - 3 = 481$$

Quotient

$$\frac{f(x+h) - f(x)}{h}$$

$$f(x) = 2x - 7$$

$$\frac{2(x+h) - 7 - (2x - 7)}{h}$$

$$\frac{2x + 2h - 7 - 2x + 7}{h}$$

$$\frac{2h}{h} = 2 = m$$

V.V.V  
IMP

Piece-wise defined functions

IMP

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 2, & \text{if } x \leq 0 \end{cases}$$

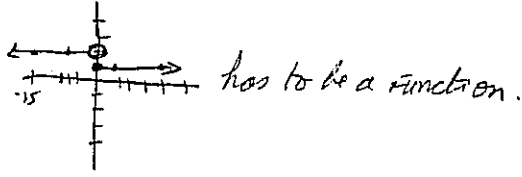
$$f(x) = 1$$

$$f(1/2) = 1$$

$$f(-15) = 2$$

$$f(0) = 1$$

$$f(-3) = 2$$



Even Functions

are symmetric w respect to y-axis

algebraically  $\rightarrow$  insert  $(-x)$  in place of  $x$  and if the result is the same then it's an even funct.  $f(x) = x^4$

$$(-x)^4 = x^4 = x^4$$

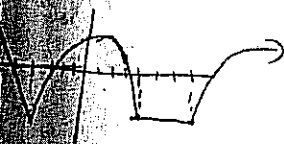
Odd Funct

are symmetric w respect to x-axis. On inserting  $(-x)$  the result is a complete opposite of the eq.  $f(x) = x^3$

$$(-x)^3 = -x^3 \neq x^3$$

If the result is not opposite or the same then the funct. is neither even or odd

Increasing & decreasing functions



dec  $(-\infty, 4]$ , inc  $[-4, 0]$ , dec  $[0, 4]$ , inc  $(4, \infty)$

Special Funct.

Constant Funct.

$$f(x) = c$$

$$R = \{c\}$$

$$D = (-\infty, \infty)$$

constant

Even Funct.

Identity Funct.

$$f(x) = x$$

$$R = (-\infty, \infty)$$

$$D = (-\infty, \infty)$$

Increasing Funct.

odd Funct.

Absolute Value Funct.

$$f(x) = |x|$$

$$R = [0, \infty)$$

$$D = (-\infty, \infty)$$

both Inc & dec Funct.

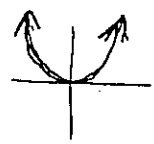
Even Funct.



Identity function turned +ve.

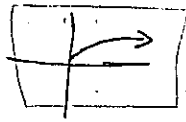
Quadratic funct.

$f(x) = x^2$   
 $D = (-\infty, \infty)$   
 $R = [0, \infty)$   
 even funct.  
 inc & dec funct.



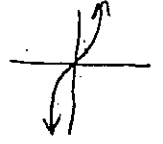
5) Square root funct.

$f(x) = \sqrt{x}$   
 $D = [0, \infty)$   
 $R = [0, \infty)$   
 increasing funct.  
 ? odd or even



6) Cube funct.

$f(x) = x^3$   
 $D = (-\infty, \infty)$   
 $R = (-\infty, \infty)$   
 inc. funct.  
 odd funct.



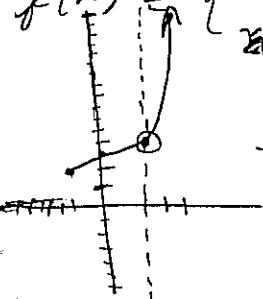
If the numerical coefficient is greater than 1 then it will narrow the sketch (towards the y-axis)  
 If the num. coeff. is between 0 & 1 then it will widen the graph (away from the y-axis)

If the num. coeff. is less than 0 then it flips the graph

Adding of a constant moves the graph up or down

A constant in the symbol of inclusion moves the graph left or right opposite the sign in the constant.

$$f(x) = \begin{cases} x+2 & \text{if } x \leq 1 \\ 2x^2 & \text{if } x > 1 \end{cases}$$



→ go a curve as  $2x^2$  is an square funct.

Algebra of functions - to perform these operations, the Domain of the 2 funct. have to be the same - MOST IMPORTANT.

1.  $(f+g)(x) = f(x) + g(x)$
2.  $(f-g)(x) = f(x) - g(x)$
3.  $(f * g)(x) = f(x) * g(x)$
4.  $(f/g)(x) = f(x) / g(x)$

TMA

$f(x) = \{(1,2), (3,6), (5,7)\}$

$g(x) = \{(2,1), (3,8), (4,9), (5,-2)\}$

$(f+g)(x) = 14 (8+6)$

$(f+g)(x) = 5(7+(-2))$

COMPOSITION functions.

$(f \circ g)(x) = f[g(x)]$

$f(x) = x^2 - 1$   
 $g(x) = 2x + 1$

$(f \circ g)(2) = f[g(2)]$   
 $= f[5]$   
 $= 48$

$f(x) = \{(1,2), (3,4), (-1,0)\}$

$g(x) = \{(2,1), (4,3), (0,-1)\}$

Inverse functions are more images of the identity line

$$m(x) = \{(3,4), (5,0), (-8,4)\}$$

$$m(x)^{-1} = \{(4,3), (0,5), (4,-8)\}$$

→ is not a function **IMP**

$$f(x) = 3x - 5$$

$$f(x)^{-1} = \frac{x+5}{3}$$

$$(f^{-1} \circ f)(x) \Rightarrow \frac{3x-8+5}{3} = \underline{x}$$

The Domain of Inverse functions is also the same like the Normal functions **IMP**

Test for Invertibility is done by a Horizontal line test → **IMP**

Identity line passes through the origin & is increasing. On folding the Identity line, the functions must overlap each other.

Hyperbola is both an Inverse & a Normal function.

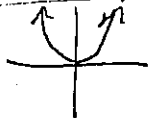
To find the Inverse funct. algebraically, shift the  $x$  & the  $y$  variables. Then solve for  $y$ . If  $f(x)$  is an inverse of  $g(x)$  we do this:

$$f[g(x)] = x \quad g[f(x)] = x \rightarrow \text{IMP} \quad \text{IMP}$$

Function in the form of  $f(x) = ax^2 + bx + c$  where  $a, b, c$  are Integers &  $a \neq 0$  is called a quadratic function (square funct.)

Vertex → is the turning point or the extreme point is also called the maximum or minimum

A quadratic eq is a square funct & so the graph is a parabola



In the eq if  $a < 0$  then the graph opens downwards resulting in a maximum pt & if  $a > 0$  then the graph opens upwards resulting in a minimum pt.

1. The vertex can be found by the formula  $\left\{ \frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right\}$  **IMP**

2. It can also be found by using the "Completing the Square" method.

$x$  intercepts can be found by substituting 0 in for the  $x$  variable & solve for  $y$  by FACTORIZATION

$x$  intercepts can also be found by using Completing the square

$x$  intercepts can be found by using the quadratic formula  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  **V.V.V. IMP**

$y$ -Intercept can be found by substituting 0 in for  $x$  in the eq

$y$ -intercepts can also be found by using synthetic division through either the maximum or the minimum point

The axis of symmetry always passes through either the maximum or the minimum point **V.V.V. IMP**

$f(x) = c \rightarrow$  constant funct.  
 $ax + bx + c = 0 \rightarrow$  linear funct.  
 $ax^2 + bx + c = 0 \rightarrow$  quad. funct.

one polynomial function, the coefficient is real  $\neq 0$  & the exponent has a non-negative integer.

$f(x) = 0 \rightarrow$  ZERO POLYNOMIAL FUNCTION

**Power functions**  $\rightarrow f(x) = ax^n$  where  $a \neq 0$  &  $n = \pm$  int. greater than one.

- a. when 'n' is even symmetry to the y-axis then its the graph is a parabola.
- b. when 'n' is odd then its symmetric to the origin & its a wiggly line.

In addition to the rest of the rules on ~~altering~~ modifying a graph, in power functions we **flatten** the graph from its bottom as the **power increases**. V.V.V. IMP

**Zeros of a function** are the **X-INTERCEPTS**. IMP

Polynomial funct. in a LINEAR FACTOR FORM  $\rightarrow (x-1/2)(x+1)(x+1)(2x+5)$

$N$  is the **degree of the polynomial**  
 $(N-1)$  is the **no. of peaks & valleys** the graph would have.

zeros  $\rightarrow 1/2, -1, -5/2$   
 multiplicity  $\rightarrow 1, 2, 1 = 4 = N =$  the deg. of the poly.  
 # of peaks & valleys  $\rightarrow N-1 = 3$   
 # zeros = deg of pol.

V.V.V. IMP

These graphs are **CONTINUOUS** curves as they never stop. IMP

**SYNTHETIC DIVISION**  $\rightarrow f(x) = x^3 + 2x^2 - 2x + 3$

**Division Algorithm**  $\rightarrow$  if  $f(x)$  &  $D(x)$  are non constant polynomial where the degree of  $f(x) \geq$  the degree of  $D(x)$ , where  $D(x) \neq 0$  then there exists unique polynomial  $q(x)$  or  $R(x)$  such that:

$$f(x) = D(x) \times q(x) + R(x)$$

divident    divisor    quotient    Remainder IMP

- the deg of  $D(x) \leq$  the deg of  $f(x)$
  - the deg of  $R(x) <$  the deg of  $D(x)$  or  $R(x) = 0$
  - the deg of  $R(x) <$   $f(x)$
- V.V.V. IMP

Synthetic division is used if we have a **LINEAR BINOMIAL DIVISOR** as  $x-h$  but not as  $mx+h$  itself. IMP

In Synthetic division, the **divisor** is the **opposite of the constant** of the original divisor. ie in  $x+3$ , the divisor is  $-3$ .

The **divident** is the numerical coefficients in the **INCREASING ORDER** of the **VARIABLE DEGREE**. -3

$$\begin{array}{r}
 -3 \ ) \ 1 \ 2 \ -2 \ 3 \\
 \underline{\phantom{-3} -3} \phantom{0} \phantom{0} \phantom{0} \\
 1 \ -1 \ 4 \ 0 \\
 \phantom{1} \underline{\phantom{-3} -3} \phantom{0} \phantom{0} \\
 \phantom{1} \phantom{-1} \phantom{4} \ 0 \ 0
 \end{array}$$

$\rightarrow$  Remainder  $\rightarrow$  IMP

V.V.V. IMP

This is a depressed Eq as the power is decreasing. ~~At the quotient~~

$(n^2 - n + 1)$  & the remainder is 0 & the answer is written as

$$\underline{(n^2 - n + 1) + 0 \cdot y} \rightarrow \text{IMP}$$

The proper form to write an answer is  $D(n) \underline{Q(n)} + R(n)$

If we have the coefficients of the dividend given like:

7 3 2 8 1

Then the power of the first term is  $\downarrow$  of the ~~quotient~~ dividend will be  $(n-2)$  where 'n' is the total # of coefficients. The power on the quotient would be  $(n-2)$ .

Remainder Theorem  $\rightarrow$  if  $f(x)$  is a polynomial funct. where degree  $n > 0$  is divided by  $(x-c)$ , then the  $f(c) = \text{Remainder}$

$(x-2)$  is a factor of the polynomial  $f(x)$  because the Remainder = 0. In other words  $2$  is a ZERO OF THE POLYNOMIAL  $f(x)$ .

Factor Theorem  $\rightarrow$  if  $f(x)$  is poly. funct. where deg of  $f(x)$ ,  $n > 0$  &  $f(c) = 0$  then  $(x-c)$  is a factor of  $f(x)$ . Conversely if  $(x-c)$  is a factor of  $f(x)$  then  $f(c) = 0$

$$f(x) = \{x^3 + 2x^2 + 13x + 10\} \text{ find } \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

$$n > 3 \quad \{n | n > 3\} \text{ or } \{n | n \geq 3\}$$

$$(3, \infty)$$

$$[3, \infty)$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } y = \frac{m}{\text{slope}} = \text{rise/run}$$

A line rising from left to right has a +ve slope  
 A line falling from left to right has a -ve slope.  
 A horizontal line has a zero slope & a  
 vertical line's slope is undefined.

$$\text{Midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{distance} = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Equation for the eqn of a circle} = (x-h)^2 + (y-k)^2 = r^2$$

Here center =  $C = (h, k)$  & Radius =  $r$

$$\text{General form of a line} = ax + by + c = 0$$

$$\text{Slope Intercept form of a line} = y = mx + b$$

$$\text{Point slope form of a line} = y - y_1 = m(x - x_1)$$

Slope Intercept & Point slope form can be transformed into the general formula for the line.

Graphing methods:

1. Two point method (3 pts)
2. Slope intercept method (2 pts)
3. x-y intercept method (2 pts)

When the slopes are equal & the y-int is different then the two lines are said to be parallel & when the slopes are the negative reciprocals of each other then they are perpendicular to each other.



August 26, 1991

Relation - a variation of the range of values of a function  
domain - a set of values of a function

Range - a set of values of a function

Domain - a set of values of a function  
Range - a set of values of a function

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DIFFERENCE QUOTIENT  $\rightarrow \frac{f(n+h) - f(n)}{h}$  where  $h \neq 0$

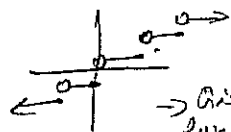
so  $f(n) = 2n - 7 = \frac{2(n+h) - 7 - (2n - 7)}{h} = \frac{2n + 2h - 7 - 2n + 7}{h} = \frac{2h}{h} = 2$

$f(n) = 2n - 7$ ; but  $f(0) = 1 - 7 = -6$ ; and  $f(1) = 2 - 7 = -5$

Piece wise defined functions  $f(n) = \begin{cases} 1, & \text{if } n \geq 0 \rightarrow 1 \\ 2, & \text{if } n < 0 \rightarrow 2 \end{cases}$

so  $f(0) = \text{bits of } 1 \therefore = 1$

$f(-0) = \text{bits of } 2 \therefore = 2$



$\rightarrow$  Given graph is a function.

EVEN Functions - are symmetric w.r.t. to the y-axis & algebraically can be found out by substituting  $-x$  in place for  $x$  & solve for  $f(x) = f(-x)$  if the result is the same as the original eq then the function is an even function.  $f(x) = x^2 \Rightarrow f(-x) = (-x)^2 = x^2 = f(x)$ .

ODD function - is graphically symmetric to the origin & has to be on both the sides of the graph of  $f \rightarrow$  not this but this  $\rightarrow f$ .

AN IDENTITY FUNCTION IS AN ODD FUNCTION. Algebraically on substituting  $-x$  for  $x$  the result should be totally opposite to the original eq.

A function w/ an envelope is an increasing function & w/ a slope is a decreasing function.


The domain for this graph is  $(-\infty, 0] \cup [0, \infty) \cap \mathbb{R} = [0, \infty)$  (to down to up)  $\rightarrow \mathbb{R}$ .


$\Psi \uparrow$   $\rightarrow$  When trying to see if the function is increasing or decreasing measure the domain to determine it.

Basic function  $\rightarrow$  1). Constant funct.  $f(x) = c \neq$   $D = (-\infty, \infty)$   
 $R = [c]$   
 constant  
 EVEN

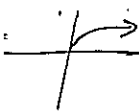
2). Identity funct.  $\rightarrow f(x) = x$   $\Psi \uparrow$   $D = (-\infty, \infty)$   
 $R = (-\infty, \infty)$   
 ODD?  
 INCREASING  $\&$   $(-\infty, \infty)$ , DOMAIN COMPRESSES

3). Square funct.  $\rightarrow f(x) = x^2$   $\Psi \uparrow$   $D = (-\infty, 0] \cup [0, \infty)$   
 $R = [0, \infty)$   
 DEC  $\rightarrow (-\infty, 0]$  INC  $[0, \infty)$   
 EVEN FUNCT.

2. Cube fund  $\rightarrow f(x) = x^3$    $D = (-\infty, \infty)$   
 $R = (-\infty, \infty)$   
 ODD FUNCT.  
 INCREASING  $(-\infty, \infty)$

1. Absolute value  $\rightarrow f(x) = |x|$    $D = (-\infty, 0] \cup [0, \infty)$   
 $R = [0, \infty)$   
 EVEN FUNCT.  
 DEC  $(-\infty, 0]$  INC  $[0, \infty)$

all linear functions but ~~one~~ power of one are straight lines on a graph & all functions of power more than one are curves.

square root fund.  $\rightarrow f(x) = \sqrt{x}$    $D = [0, \infty)$   
 $R = [0, \infty)$   
 NEITHER EVEN NOR ODD  
 INC  $\rightarrow [0, \infty)$

-ve coefficient flip the graph

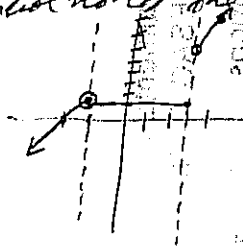
coefficient greater than 1 narrow by x-axis

coefficient less than 1 widen the graph by x-axis

adding a constant goes up or down the no. of places

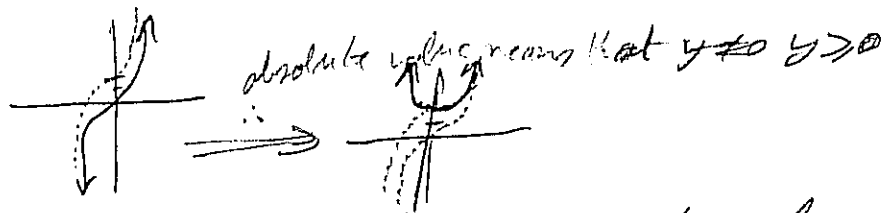
constant under a index or symbol moves the place opposite the sign of the constant either left or right side.

$$f(x) = \begin{cases} x^2, & x > 3 \\ 2, & -1 < x \leq 3 \\ x+2, & x \leq -1 \end{cases}$$



$$g(x) = |x^3 + 2|$$

IMP



leave fractional answers simplified in one fraction only.

do algebra of functions, the domain of the 2 functions should be the same. eg  $f(x) = x^2 - 9 \rightarrow D = \text{all } \mathbb{R}$  &  $g(x) = 2x + 3 \rightarrow D = \text{all } \mathbb{R}$  so

could be worked on. eg  $\rightarrow f(x) = \{(4, -5), (3, -6), (5, -4)\}$

$$g(x) = \{(-3, -3), (3, 9), (4, -2), (5, -2)\}$$

$$\text{so now } f(x) + g(x) = 8 + 6 = 14 \quad 9 + (-2) = 7$$

$$1) (f+g)(x) = f(x) + g(x)$$

$$2) (f-g)(x) = f(x) - g(x)$$

$$3) (f \times g)(x) = f(x) \cdot g(x)$$

$$4) (f/g)(x) = f(x)/g(x) \text{ where } g(x) \neq 0$$

$$* \text{ COMPOSITION FUNCTION } \rightarrow (f \circ g)(x) = f[g(x)] \text{ or } (g \circ f)(x) = g[f(x)]$$

$$f[g(x)] \rightarrow (f \circ g)(x)$$

$$g[f(x)] \rightarrow (g \circ f)(x)$$

$$* \text{ NOTE } \rightarrow \frac{6x^2 + 11x + 3}{-7x^2 + 37x - 10} = \frac{6x^2 + 11x + 3}{(-7x - 2)(x + 5)} \text{ where } x \neq \frac{2}{7}; -5; \frac{1}{3}$$

DO NOT FORGET TO MENTION THE 2 EXCEPTIONS IN DIVISION OF FRACTION

$$f(x) = \{(2, 2), (3, 4), (-1, 0)\} \rightarrow \text{both are functions.}$$

$$f^{-1}(x) = \{(2, 2), (4, 3), (0, -1)\}$$

Both the relations are called inverse functions.

$$m(x) = \{(3, 4), (5, 6), (-8, 9)\} \rightarrow \text{funct.}$$

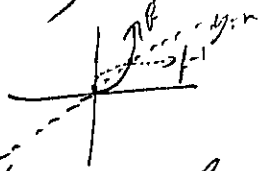
$$m^{-1}(x) = \{(4, 3), (6, 5), (9, -8)\} \rightarrow \text{not a funct.}$$

$\rightarrow$  A function is said to be invertible if the relation formed by interchanging the 'y' & 'x' of each ordered pair of the given function is a function.

If  $f(x)$  is a function then  $f^{-1}$  is the inverse function.

DOMAIN OF THE INVERSE FUNCTION ALSO HAS ALL REAL NOS. WITH 2 EXCEPTIONS  
 Test for invertibility can be done w/ the Horizontal Line

$$f(x) = x^2; x \geq 0 \rightarrow \text{MIND THE EXCEPTION.}$$

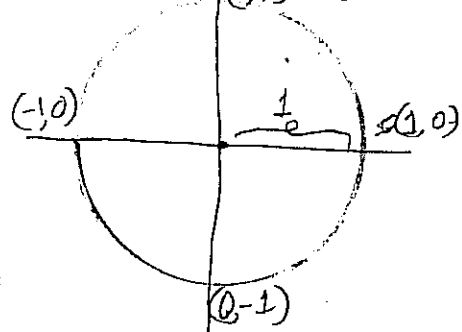


where the orig. funct. graph crosses the identity line the inverse funct. graph also crosses the same point.  
 Hyperbola in both inverse & normal funct. P.T.O

Trig functions are real nos based on a circle or unit circle.

Developed by Hipparchus.

Unit Circle  $x^2 + y^2 = 1$ ; normal circle  $(x-h)^2 + (y-k)^2 = r^2$



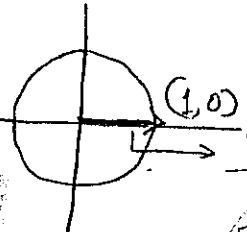
Circumference of a unit circle =  $2\pi r$

**IMP**

Length of subtended arc at an  $\theta$  of  $25^\circ$  rad =  $2.5 \times \frac{\pi}{180} = 0.436$

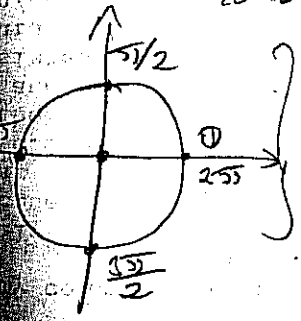
As wrapping functions, one rotation on a number line is =  $6.28$   
 & two rotations is =  $12.56$  units.

+ sine reals are generated in a clockwise rotat. & + sine reals are generated in a counterclockwise rotation.

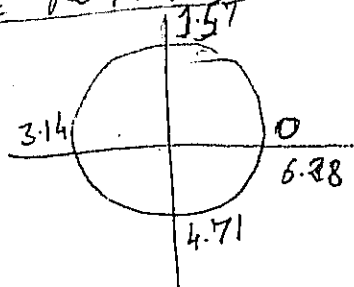


"Initial Ray" -> always starts at +ve x-axis.

Rotation of initial ray towards any other direction calls it a "TERMINAL RAY"  
POSITIVE ROTATION



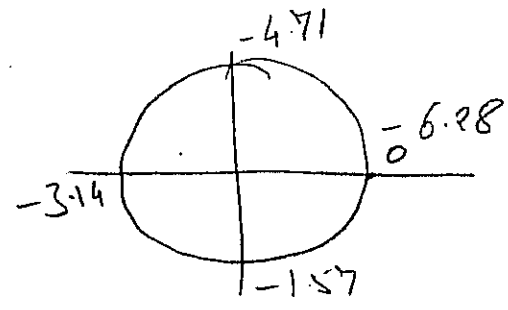
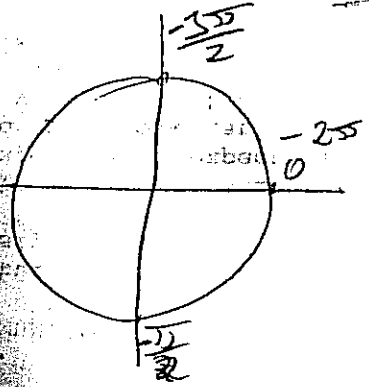
RADIAN measures,



REAL measures

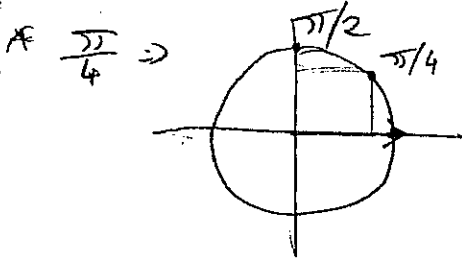
**IMP**

NEGATIVE ROTATION



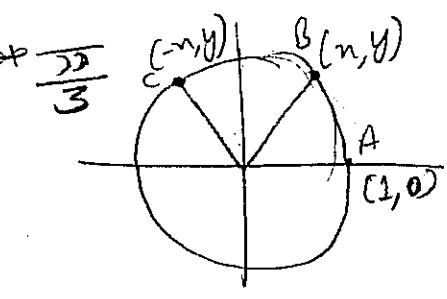
\*  $W(0) = (1, 0)$   
 $W(\frac{\pi}{2}) = (0, 1)$   
 $W(\pi) = (-1, 0)$   
 $W(\frac{3\pi}{2}) = (0, -1)$   
 $W(2\pi) = (1, 0)$

\*  $\sin t = y$   
 cosine  $t = x$   
 tangent  $t = \frac{y}{x} \neq 0$   
 cosecant  $t = \frac{1}{y} \neq 0$   
 secant  $t = \frac{1}{x} \neq 0$   
 cotangent  $t = \frac{x}{y} \neq 0$



$x^2 + y^2 = 1$   
 2 coordinates must be the same  
 $x^2 + y^2 = 1 \Rightarrow x = \frac{\sqrt{2}}{2}$   
 $x^2 + x = 1$   
 $2x^2 = 1$   
 $x^2 = 1/2$

$\frac{\pi}{4} = 45^\circ = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$



located in 2nd quad.

$AB = BC$   
 $\therefore AB = BC$

$= (\sqrt{(1-n)^2 + (0-y)^2}) = (\sqrt{(n+n)^2 + (y-y)^2})$   
 $= (1-n)^2 + y^2 = 4n^2$   
 $1 - 2n + n^2 + y^2 = 4n^2$   
 $1 - 2n + y^2 = 3n^2$   
 solve for  $y^2$  & sub. in  $x^2 + y^2 = 1$   
 $1 - 2n = 1 - n^2$

$\frac{\pi}{3} = 60^\circ = (\frac{1}{2}, \frac{\sqrt{3}}{2})$

$1 - 2n + 1 - n^2 = 3n^2$   
 $0 = 4n^2 + 2n - 2$   
 $0 = 2n^2 + n - 1$   
 $0 = (2n-1)(n+1)$   
 $n = 1/2, *$

$b^2 = 1 - 1/4 = 3/4$   
 $b = \pm \frac{\sqrt{3}}{2}$

Is my communication: \_\_\_\_\_  
 timely \_\_\_\_\_  
 concise \_\_\_\_\_  
 clear \_\_\_\_\_  
 effective \_\_\_\_\_  
 to the right \_\_\_\_\_  
 audience \_\_\_\_\_  
 consistent \_\_\_\_\_  
 accurate \_\_\_\_\_  
 open/candid \_\_\_\_\_  
 constructive \_\_\_\_\_

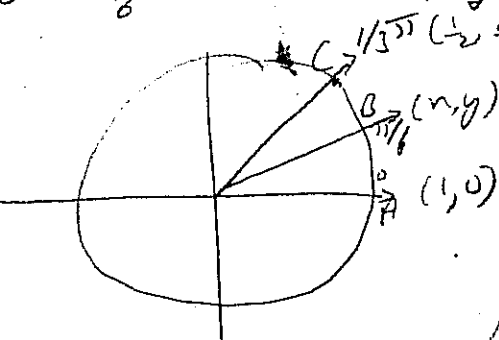
1 = far exceeds your expectations  
 2 = exceeds your expectations  
 3 = meets your expectations  
 4 = needs improvement  
 5 = unsatisfactory

Keeping in mind the stated intent of the communications:  
 A = to inform or explain  
 B = to enroll or enlist  
 C = to request or command

Please rate my communications using a scale of:

QUEST COMMUNICATIONS EFFECTIVENESS SURVEY - Bill Scheerer, Round 1  
 Please return to Bill Scheerer at hogaxjws or at mailjwscheerer, or by paper (anonymously is fine)

$\theta = \frac{\pi}{6}$  located in 1st quad.



If  $\theta$  IF THE ARCS ARE EQUAL THEN THE SEGMENTS ARE EQUAL TOO

$$\left( \sqrt{(x-1)^2 + y^2} \right)^2 = \left( \sqrt{\left(x-\frac{1}{2}\right)^2 + \left(y-\frac{\sqrt{3}}{2}\right)^2} \right)^2$$

$$= (x-1)^2 + y^2 = \left(x-\frac{1}{2}\right)^2 + \left(y-\frac{\sqrt{3}}{2}\right)^2$$

$$= x^2 - 2x + 1 + y^2 = x^2 - x + \frac{1}{4} + y^2 - \sqrt{3}y + \frac{3}{4}$$

$$\therefore -x + 1 = -\sqrt{3}y + \frac{3}{4}$$

$$x = 1 - \sqrt{3}y$$

$$x^2 + y^2 = 1$$

$$3y^2 + y^2 = 1$$

$$y^2 = \frac{1}{4}$$

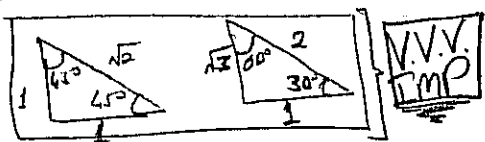
$y = \pm \frac{1}{2}$  in first quad, so  $\frac{1}{2}$

$$x = 1 - \sqrt{3}\left(\frac{1}{2}\right)$$

$$x = \frac{2 - \sqrt{3}}{2}$$

$x = \frac{\sqrt{3}}{2}$  & true as in 1st quad

$\frac{\pi}{6} = 30^\circ = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



TRIG IDENTITIES

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$$

To find if a given coordinate is the coordinate of a unit circle simply add the squares of the x & y coordinate & if the sum is equal to 1, then it is a unit circle.

The circumference of a unit circle is =  $2\pi r = 2\pi = 360^\circ$

COTERMINAL ANGLE  $\rightarrow$  angles that start with the same initial ray & have the same terminal ray.

eg  $\frac{5\pi}{4} = \frac{7\pi}{4} = \frac{9\pi}{4}$  or  $160^\circ = 510^\circ = 870^\circ = -20^\circ = -570^\circ$

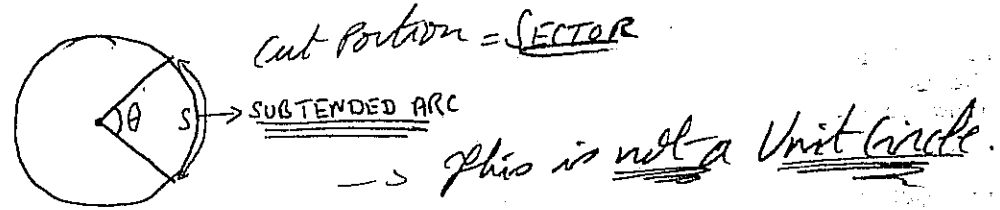
$1^\circ = 60 \text{ mins}$   
 $1 \text{ mins} = 60 \text{ SEC}$

\* CONVERSIONS  $\rightarrow 28.37^\circ$   
 $= 28^\circ (22.2) (12)$   
 $= 28^\circ 22' 12''$

\*  $72^\circ 15' 17''$   
 $72^\circ \frac{17}{60} = 0.28\bar{3}$   
 $72^\circ \frac{15 + 0.28\bar{3}}{60} = 0.26$   
 $= 72.26^\circ$

Convert rad. to deg. & deg. to rad.

\*  $\frac{3\pi}{2} \times \frac{180}{\pi} = 270^\circ$   
 \*  $90^\circ \times \frac{\pi}{180} = \frac{\pi}{2}$



\* central  $\theta \rightarrow$  is an  $\angle$  with the vertex at the center of an  $\angle$ .

\* Length of the subtended arc =  $S = r \cdot \theta$   $\rightarrow$  IMP

\* Area of a sector =  $\frac{1}{2} r^2 \theta$  or  $\frac{1}{2} r \cdot s$   $\rightarrow$  IMP

*theta should be converted to Radians*

Trig functions with a radius other than 5/1.

$\sin \theta = \frac{y}{r}$      $\csc \theta = r/y$   
 $\cos \theta = x/r$      $\sec \theta = r/x$   
 $\tan \theta = y/x$      $\cot \theta = x/y$

Trig functions on a RIGHT A

$\sin \theta = \text{opp/hyp}$      $\csc \theta = \text{hyp/opp}$   
 $\cos \theta = \text{adj/hyp}$      $\sec \theta = \text{hyp/adj}$   
 $\tan \theta = \text{opp/adj}$      $\cot \theta = \text{adj/opp}$

Some people have curly brown hair turning permanently black

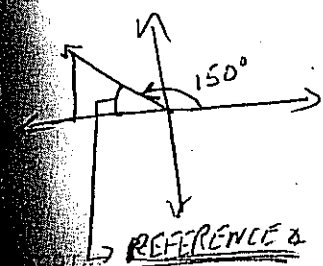
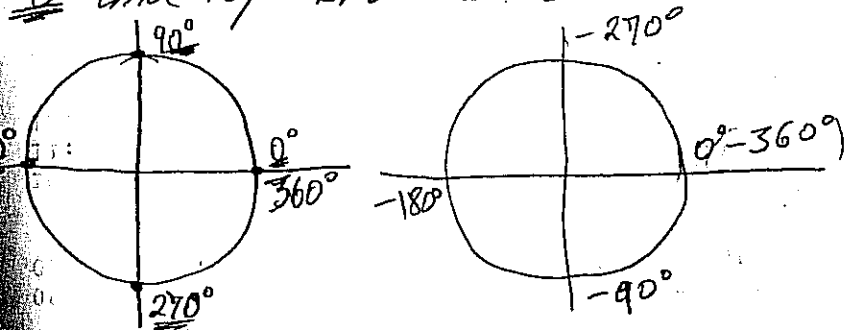
$\frac{P}{H} = \frac{O}{H} = \frac{T}{S}$



ALL STUDENTS TAKE CALCULUS

II S' sine & cosec +ve	I All are +ve	V.V.V. IMP
III tan & cot are +ve	IV cos & sec are +ve	

$\theta$  will represent DEGREE measure &  $\theta$  will represent RADIAN measure.



REFERENCE angle  $\rightarrow$  is a acute angle & is formed by a line from the terminal ray  $\perp$  to the x-axis.

\* In the II<sup>nd</sup> quad,  $\text{ref } \alpha = 180^\circ - \text{given } \alpha$ .

\* In the III<sup>rd</sup> quad,  $\text{ref } \alpha = \text{given } \alpha - 180^\circ$ .

\* In the IV<sup>th</sup> quad,  $\text{ref } \alpha = 360^\circ - \text{given } \alpha$ .

THERE ARE MORE THAN ONE ROTATIONS THEN USE -

$P(95^\circ) = \dots$  has to be a multiple of 255  $\rightarrow$  IMP

$P(\theta) = (\cos \theta, \sin \theta)$

$0 < \angle < 255$   
 $0 < \angle < 255$   
 $0 < \angle < 360^\circ$  } will always be a true  $\alpha$  or rad. measure } IMP

$\text{eg: } P\left(\frac{4155}{4}\right) = P\left(\frac{15}{4}, \frac{4055}{4}\right) \rightarrow \text{I}^{\text{st}} \text{ quad} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$P(-675^\circ) = (45^\circ + (-720^\circ)) \rightarrow \text{I}^{\text{st}} \text{ quad} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\cos^2 \theta = \cos^2 \theta$

$\sin^2 \theta = \sin^2 \theta$

$\tan^2 \theta = \tan^2 \theta$

PYTHAGOREAN IDENTITIES

1.  $\sin^2 \theta + \cos^2 \theta = 1$

2.  $1 + \tan^2 \theta = \sec^2 \theta$

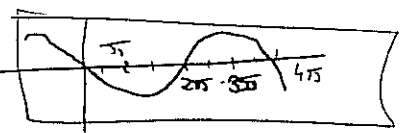
3.  $\cot^2 \theta + 1 = \csc^2 \theta$

V.V.V. IMP

24, 25

$\sin(-90^\circ) = -1$   
 $-\sin 90^\circ = -1$

**IMP PAGE**



$\sin(-\theta) = -\sin \theta$ , odd

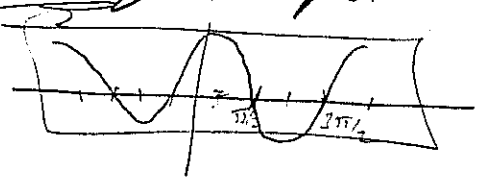
sine is a continuous funct. & is odd & symmetric to the origin.

$\cos(-45^\circ) = \frac{\sqrt{2}}{2}$

$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$\cos(-\theta) = \cos \theta$ , Even

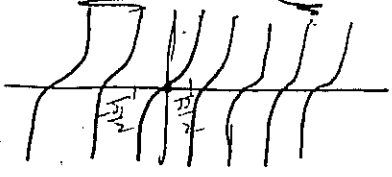
cos is a continuous funct. & is even, symmetry w/ respect to y-axis



$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin \theta}{\cos \theta}$

$\tan(-\theta) = -\tan \theta$ , odd

Has asymmetry with respect to the origin & is odd



main of Funct.  $\rightarrow$

$\sin t$  &  $\cos t = \{t : t = \text{real}\}$

$\tan t = \frac{\sin t}{\cos t}$ ,  $\sec t = \frac{1}{\cos t}$

$D = \{ \text{real} \mid \cos t \neq 0 \}$   
 $\{ t \mid t \neq \frac{\pi}{2} + n\pi \}$   
 $\rightarrow$  Integer

$\cot t = \frac{\cos t}{\sin t}$ ,  $\csc t = \frac{1}{\sin t}$

$D = \{ \text{real} \mid \sin t \neq 0 \}$   
 $= \{ t \mid t = n\pi \}$   
 $\rightarrow$  Real #

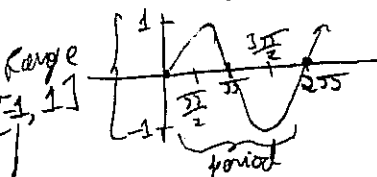
**V.V.V. IMP**

**V.V.V. IMP**

AS FUNDAMENTAL PERIOD IN A SINE CURVE IS FROM 0 To  $2\pi$ .

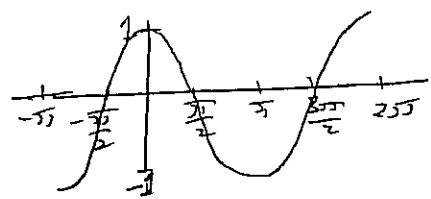
called a complete cycle.

A sine curve is an ~~even~~ <sup>continuous</sup> odd curve with a period of  $2\pi$  but since it is odd, it intersects x-axis at  $0$  & at the point  $\pi$  its Range is  $[-1, 1]$  its Domain is all reals



ALWAYS GO FROM BOTTOM TO TOP.

\* A cosine curve is an even continuous funct. with a period of  $2\pi$  but since it is even, it intersects x-axis at  $0$  &  $2\pi$  its Range is  $[-1, 1]$ . its Domain is all reals.



\* Gen Formas

$f(x) = a \sin(kx + b)$

$g(x) = a \cos(kx + b)$



|a| determines the amplitude of the sine or cosine curve & thus changing the RANGE

- a). If  $a > 1$  then there is a vertical stretch
- b). If  $0 < a < 1$ , then vertical shrink
- c). If  $a < 0$ , then FLIP THE CURVE

Please get back to me to confirm your attendance.

John Balasare  
arch3:jb4  
908-949-6205

1. Take 287 North to Exit 31  
2. Exit 31 is South Street Morristown  
3. Turn Right at traffic light at end of ramp  
4. Proceed about 1 mile  
5. Entrance to Belcore in on the right  
6. White Brick Building - Building #2

To reach the Belcore building from the Homdel area:

Thursday, May 28, 1992  
9:00 am  
Belcore Building #2  
445 South Street Morristown

A demo of SuperBook has been scheduled for:  
Re: Online Documentation Standards Members  
Re: SuperBook Demo in Morristown

From arch3:jb4 Mon May 18 11:16 EDT 1992  
To: alataf@cc, arch2@dnn, bct, deronda, jb4, lpb, atmali@ckhoffman, atmali@ebtron, atmali@joanneb, atmali@lafey, atmali@mcarey, atmali@midddaugh, atmali@tronalides, drdo@imary, druks@ouray, druks@wolpert, tjtlc@allamb, homxb@sball, homkx@lagan, homqax@acj, hostar@kpd, thlpw@straka, thlpy@napatch, hndy@tc@gll, mtdcb@pdn@j.new, mtdcb@pdn@k, fitzgerald, mtme@cml, mttml@mal, mtgzy@erk, mtunp@jlm, mtunp@nee, mtuxo@lhl1, nwwpa@wjb, techman@trb, uhura@lln, uhura@nam, vtoln@gpp, wrddo@cml, wrddo@lgj

Subject: SuperBook Demo  
Status: RO

$k'$  gives the horizontal changes

$k'$  changes the period of the curve =  $\frac{2\pi}{|k|}$  **V.V. IMP**

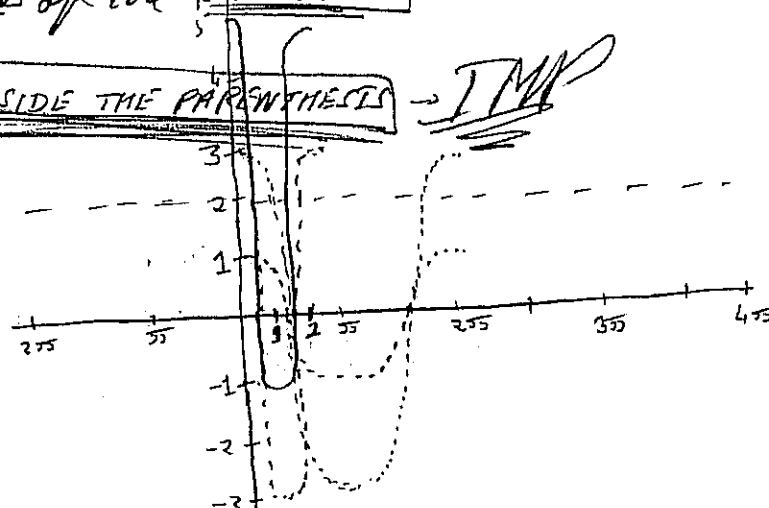
The  $(a \sin(bx + c) + d)$  in **PARENTHESES** gives you the **PHASE SHIFT**.

=  $a \sin(bx + c) + d$   $\left[ \frac{2\pi}{b} \right]$  where the curve } **IMP**  $0 \leq bx + c < 2\pi$   
 ↳ where the curve begins }  
 ↳ where the curve ends. }

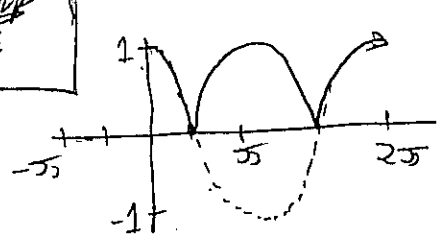
The addition or subtraction of a **CONSTANT** moves the x-axis **UP** or **DOWN** depending on the **SIGN** of the **CONSTANT**.

**THE CONSTANT IS ALWAYS OUTSIDE THE PARENTHESES** → **IMP**

$y = 2 \sin 2 + 3 \cos 5\pi x$   
 $\frac{2\pi}{5\pi} = 2$   
 $A \rightarrow R = [-1, 1]$   
 Per = 2  
 $D = R's$



$y = |\cos 2x|$  → **IMP**

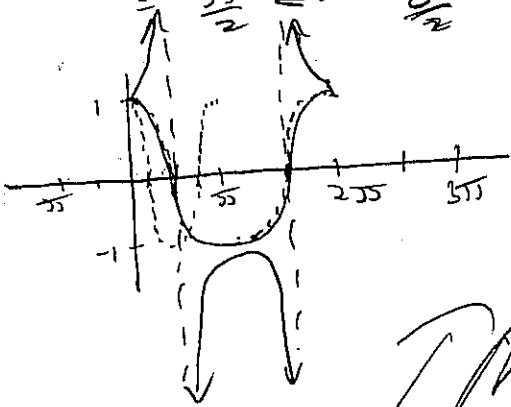


with the  $\cos$  &  $\sec$  curves, first graph the  $\sin$  &  $\cos$  curves carefully & then sketch the  $\tan$  or  $\sec$  curves with the help of **ASYMPTOTES** → the place where the curve hits the x-axis.

**V.V.V IMP**  
 $\sec$  curves have no amplitude & only vertical asymptotes & range changes.  
 \* **NO AMPLITUDE** → **IMP**

$$y = \sec(-2x - \pi)$$

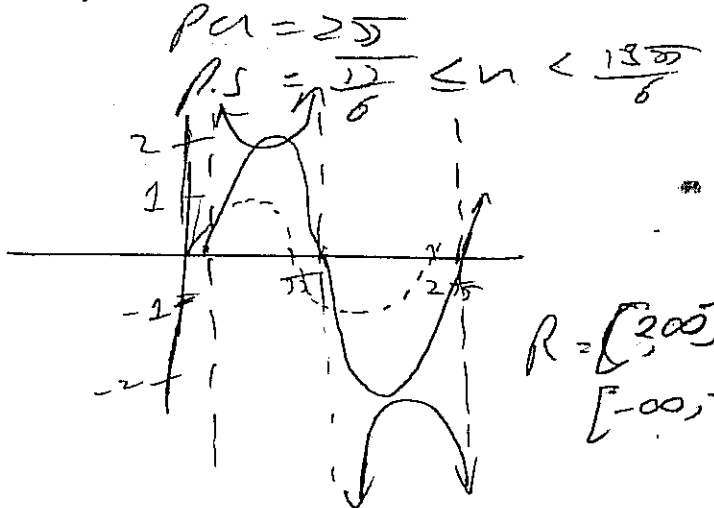
$P.O.C = \pi$   
 $P.S = 0 < 2x - \pi < 2\pi$   
 $= \frac{\pi}{2} < x < \frac{3\pi}{2}$



~~$$* y = \sec\left(\frac{n + \pi}{6}\right)$$~~

$$* y = 2 \cos\left(n - \frac{\pi}{6}\right)$$

$$y = 2 \sin\left(n - \frac{\pi}{6}\right)$$



$R = [200]$   
 $[-\infty, -$

IF  $y = \sin\left(-n + \frac{\pi}{2}\right)$   
 odd  $\rightarrow y = \cos\left(-n + \frac{\pi}{2}\right)$

THEN  $y = -\sin\left(n - \frac{\pi}{2}\right)$   
 $y = -\cos\left(n - \frac{\pi}{2}\right)$

IF  $y = \cos\left(-n + \frac{\pi}{2}\right)$   
 even  $\rightarrow y = \sec\left(-n + \frac{\pi}{2}\right)$  THEN  $y = \cos\left(n - \frac{\pi}{2}\right)$   
 $y = \sec\left(n - \frac{\pi}{2}\right)$

Those who are subscribing to our LAN (Marc, John, and me so far, Jim is next) can get their mail from the LAN without logging on to homxpd thereby saving money. The LAN server, hostar connects to the Datakit network and gets my mail from homxpd a present. - KP



From: kpd  
 Date: Wed May 20 11:24:16 EDT 1992  
 UA-Content-ID: <PMX-LAN-2.1-\*\*\*\*\*-hostar-kpd-791>  
 End-of-Header:  
 Email-Version: 2  
 Phone: 908-949-9175  
 UA-Message-ID: <wlnPMXSTAR-2.1b-kpd-xyxyxyxy-10>  
 To: homxpd!sgbal  
 In-Reply-To: your message of Mon May 18 12:03 EDT 1992  
 End-of-Protocol:  
 Content-Type: Text  
 Content-Length: 1036  
 Status: R

From: hostar!kpd Wed May 20 11:18 EDT 1992  
 Message-Version: 2  
 >To: homxpd!sgbal

Graph the tan & cot curve.

The tan & cot curve is an odd funct. symmetric to the origin.  
 The RANGE is  $[-\infty, \infty]$ . The Domain is  $\{R = \text{nt. } \cos \neq 0\}$

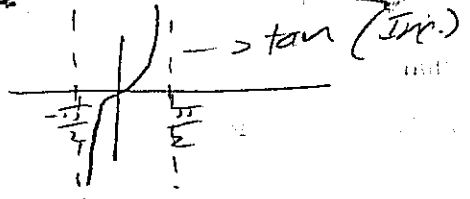
The fundamental PERIOD =  $0$  to  $\pi$ . IMP

The formula for finding the bc. of tan & cot curves is =  $\frac{\pi}{k}$

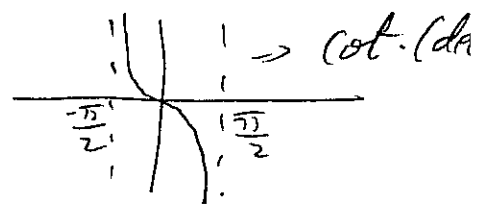
The phase shift =  $-\frac{\pi}{2} < kx + b < \frac{\pi}{2}$  IMP  
 $\hookrightarrow \text{no } \leq \rightarrow \text{IMP}$

TANGENT is an INCREASING FUNCT. & COTANGENT is a DECREASING FUNCT.

has no amplitude IMP



- If  $a > 1$ , then it thins the curve.
- If  $0 < a < 1$ , then it widens the curve.
- If  $0 > a$ , then the curve is flipped



$$y = \tan\left(\frac{x}{2} - \frac{\pi}{3}\right)$$

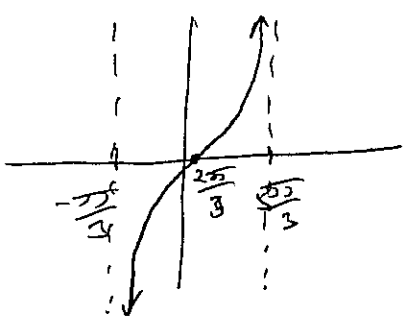
$$\text{Per} = 2\pi \quad \{R\} \quad \{C \neq 0\}$$

$$D = \{R\} \quad \{C \neq 0\}$$

$$R = [-\infty, \infty]$$

$$P.S. = -\frac{\pi}{2} < \frac{x}{2} - \frac{\pi}{3} < \frac{\pi}{2}$$

$$= -\frac{\pi}{2} < x < \frac{5\pi}{3}$$



IMP THE ZERO'S of a curve are the points on the x-axis where the curve intersects.

$$\text{Per} = 2\pi$$

$$\therefore \text{zero} = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$$

----- Begin Original Message -----

Message-Version: 2

From: homxb!sgbali

Date: Mon May 18 12:03 EDT 1992

Received: from homxb by hostar.ho.att.com; Mon, 18 May 1992 12:02 EDT

End-of-Header:

Email-Version: 2

To: homxb!kpd

End-of-Protocol:

Content-Type: text

Content-Length: 475

KP,

Are we all moving to a new system?

Shri

>From kpd Fri May 15 11:19 EDT 1992

Status: RO

NOTICE: The login 'homxb!kpd' which is owned by k.p.das has been moved to 'hostar!kpd'. Your mail has been automatically forwarded. This mail forwarding service will only be available for a finite length of time, so please note the new address and begin using it as soon as possible.

Thank you,

HOCC UNIX Support Group

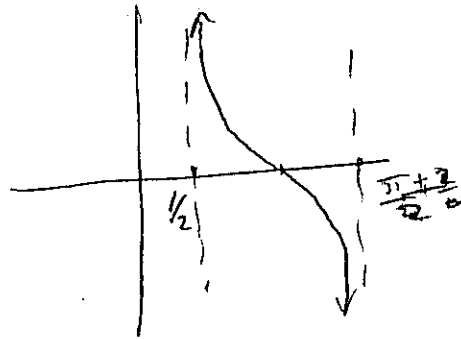
\*  $h(n) = \cot(2n-1)$

$$p.u. = \frac{\pi}{2}$$

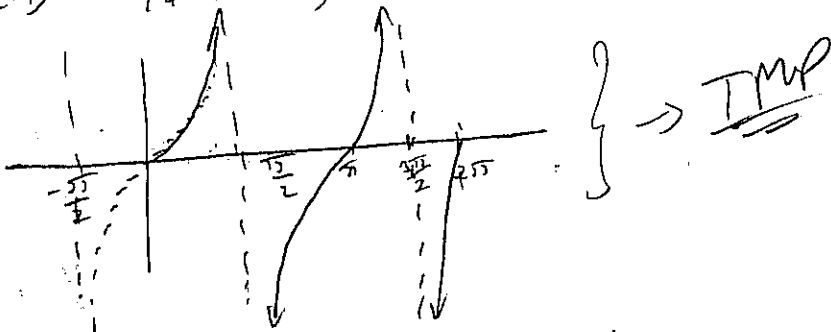
$$p.s. = -\frac{\pi}{2} < 2n-1 < \frac{\pi}{2}$$

$$\frac{1}{2} < n < \frac{\pi+1}{2}$$

$$\text{zero} = \frac{\pi}{4} + \frac{1}{2}$$



\*  $f(n) = \tan n$ , interval  $[0, 2\pi]$



If  $y = \tan(-n + \frac{\pi}{2})$ , then  $y = -\tan(n - \frac{\pi}{2})$

If  $y = \cot(-n + \frac{\pi}{2})$ , then  $y = -\cot(n - \frac{\pi}{2})$

IMP

## FUNDAMENTAL IDENTITIES

### Pythagorean Identities

1)  $\sin^2 \theta + \cos^2 \theta = 1$

2)  $1 + \tan^2 \theta = \sec^2 \theta$

3)  $\cot^2 \theta + 1 = \csc^2 \theta$

### Odd-Even Identities

a)  $\sin(-t) = -\sin t$

b)  $\cos(-t) = \cos t$

c)  $\tan(-t) = -\tan t$

d)  $\csc(-t) = -\csc t$

e)  $\sec(-t) = \sec t$

f)  $\cot(-t) = -\cot t$

### Reciprocal Identities

a)  $\sin \theta = \frac{1}{\csc \theta}$       $\csc \theta = \frac{1}{\sin \theta}$

b)  $\cos \theta = \frac{1}{\sec \theta}$       $\sec \theta = \frac{1}{\cos \theta}$

c)  $\tan \theta = \frac{1}{\cot \theta}$       $\cot \theta = \frac{1}{\tan \theta}$

d)  $\cot \theta = \frac{\cos \theta}{\sin \theta}$       $\tan \theta = \frac{\sin \theta}{\cos \theta}$

### HALF-ANGLE FORMULAS

$\sin \frac{1}{2} t = \pm \sqrt{\frac{1 - \cos t}{2}}$

$\cos \frac{1}{2} t = \pm \sqrt{\frac{1 + \cos t}{2}}$

$\tan \frac{1}{2} t = \pm \sqrt{\frac{1 - \cos t}{1 + \cos t}}$

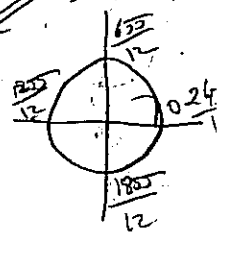
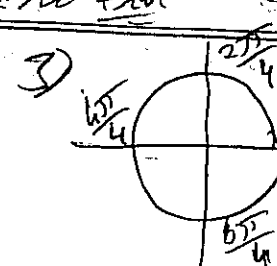
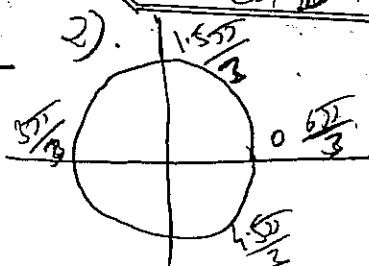
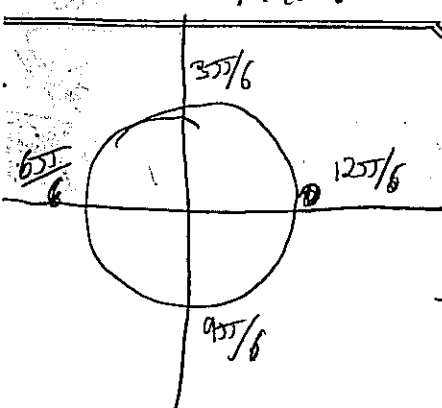
**IMP**  
The sign or -ve sign will be determined by what quadrant the graph falls in

eg  $\sin \frac{1}{2} (4\pi) = \sin 2\pi = \text{III}^{\text{rd}}$  quad & no -ve

$\cos \frac{5\pi}{12} = \cos \frac{1}{2} (\frac{5\pi}{6})$

$\rightarrow$  In II quad. & no +ve

Take the  $\frac{1}{2}$



## Sum & Difference Formulas

$\cos(t+s) = \cos t \cos s - \sin t \sin s$

$\cos(t-s) = \cos t \cos s + \sin t \sin s$

$\sin(t+s) = \sin t \cos s + \cos t \sin s$

$\sin(t-s) = \sin t \cos s - \cos t \sin s$

$\tan(t+s) = \frac{\tan t + \tan s}{1 - \tan t \tan s}$

$\tan(t-s) = \frac{\tan t - \tan s}{1 + \tan t \tan s}$

If given a single term such as  $\tan 195^\circ$

**IMP**  
try to break it into something with the denominator of 34 or 12

### DOUBLE-ANGLE FORMULAS

1)  $\cos 2t = \cos^2 t - \sin^2 t$

a)  $\cos 2t = 1 - 2\sin^2 t$

b)  $\cos 2t = 2\cos^2 t - 1$

2)  $\sin 2t = 2\sin t \cos t$

3)  $\tan 2t = \frac{2\tan t}{1 - \tan^2 t}$



From jolang Tue May 26 16:02 EDT 1992

From: hlwpj!jolang (Joan H Lang +1 908 949 0286)

To: hlwpj!diane, hlwpj!jolang, hlwpj!vsh, homxb!fine, homxb!jtupino, homxb!khn, homxb!norris, homxb!pav, homxb!sgbali, homxb!tlr, homxc!darla, hostar!kpd, hostar!march

Cc: attmail!nshaer (Norman R Shaer), mtdcc!vbl (Victor B Lawrence)

Subject: Friday, May 29

Status: RO

Following is the tentative agenda for the meeting on May 29, 1992 with Norm Shaer at 9 a.m.

- 9 am to 10:30 am - QUEST presentation by N. Shaer
- 10:30 am to 11:00 am - Brief overview of projects by Shri and Tom with descriptions by

Marc Hornby - Performance Support for TCC NESAC and

Jim Tupino - ISDN Videophone Technical Trial

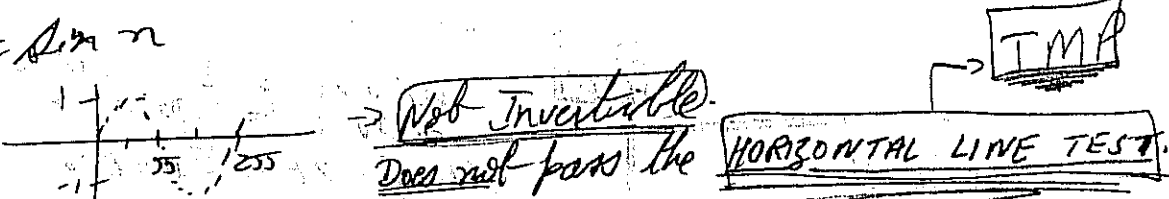
The meeting is in Room 3N-418. Immediately following the meeting, we will leave for the boat ride/luncheon in Atlantic Highlands.

Joan Lang  
hlwpj!jolang

# IMP INVERSE TRIG FUN

\*  $f(n) = \sin n \rightarrow$  periodic fund.  
 $f(n) = \sin n \rightarrow$  Restricted Domain & Invertible

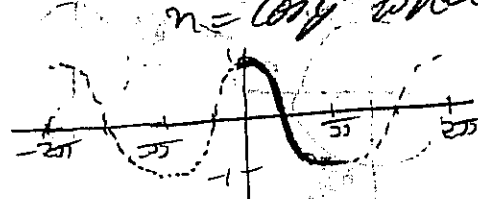
\*  $f(n) = \cos n \rightarrow$  periodic  
 $f(n) = \cos n \rightarrow$  Rest. Domain & Invertible.

\*  $f(n) = \sin n$   
  
 Not Invertible. Does not pass the HORIZONTAL LINE TEST. IMP

\*  $f^{-1}(n) = \cos^{-1} n = \text{Arccos } n$

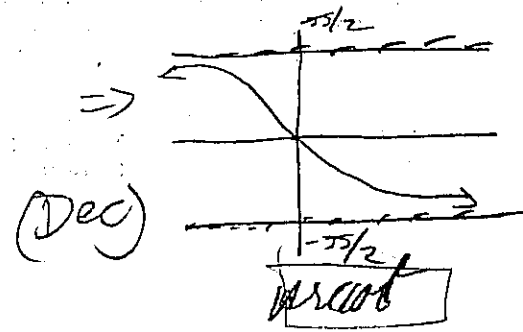
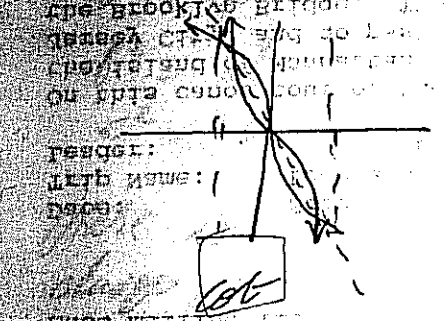
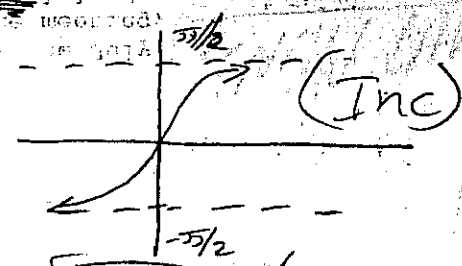
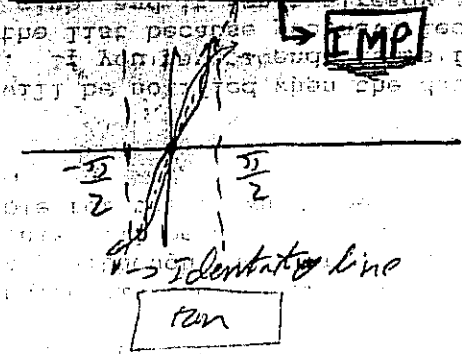
\* defn: Arcsin funct. is denoted by  $y = \arcsin n$  or  $y = \sin^{-1} n$   
 if  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  or  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$  or in quad I & IV IMP

\* defn: Arccos is denoted by  $y = \arccos n$  or  $y = \cos^{-1} n$  iff  
 $n = \cos y$  where  $0 \leq y \leq \pi$  which is quad I & II IMP



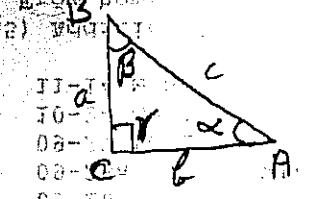
Defn: Inverse of  $\tan^{-1} x$  is denoted by  $\tan^{-1} x$  or  $\tan^{-1} x$  off  $\tan^{-1} x$  where  $\frac{-\pi}{2} < y < \frac{\pi}{2}$  on quad I & IV. IMP

$f(x) = \tan(x)$   
 $D = \left[ \frac{-\pi}{2}, \frac{\pi}{2} \right]$

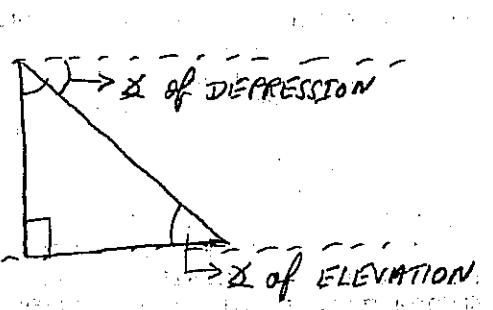


**MIND THE QUADRANTS**

Treat  $\arccos$  &  $\arcsin$  as  $\arccos$  &  $\arcsin$  respectively. IMP  
 for any inverse funct. quad. III is not at all needed.



$\sin \alpha = \frac{a}{c}$        $\csc \alpha = \frac{c}{a}$   
 $\cos \alpha = \frac{b}{c}$        $\sec \alpha = \frac{c}{b}$   
 $\tan \alpha = \frac{a}{b}$        $\cot \alpha = \frac{b}{a}$



line of sight to the object =  $\alpha$  of depression

Gray wire  $\rightarrow$  Hypotenuse

**LAW OF SINES** can be used for ANY TYPE OF A TRIANGLE

when 2  $\alpha$  and any side of a  $\Delta$  are given, they always yield a unique  $\Delta$   
 when 2 sides and a  $\alpha$  opposite is given, then it yields an AMBIGUOUS CASE

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$
 IMP

Given two sides and ~~opposite~~ the sides. There are 3 possibilities of a  $\Delta \Rightarrow 1 \Delta$ ,  $2 \Delta$  or  $no \Delta$

LAW OF COSINES

$\rightarrow$  is used when -

- 1) 2 sides and an included  $\angle$  are given.
- 2) 3 sides are given.

KNOW LA OF SINES & COSINES

$$a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$$

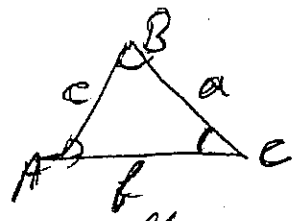
$$b^2 = a^2 + c^2 - 2ac \cdot \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos \gamma$$

V.V.V. IMP

There is no ambiguous case in law of cosines & will always generate a  $\Delta$ .

HERON'S FORMULA



area  $\Delta = \frac{1}{2} b \times \text{alt}$   
 Peri  $\Delta = a + b + c$

area of a  $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$   
 where  $s = \frac{a+b+c}{2}$

V.V.V. IMP

POLAR COORDINATES

In this system, center is the pole based upon the ordered pair  $(r, \theta)$ .  
Polar axis  $\rightarrow$  starting axis on  $0^\circ$ .  
 $r \rightarrow$  Radius &  $\theta = \angle$  of rotation from the initial ray to the terminal ray.

IF THERE IS A NEGATIVE RADIUS THEN  $\rightarrow$

$(-r, \theta) \Rightarrow (|r|, \theta \pm 180^\circ)$

If the  $\theta$  rotation is counter clockwise then add  $180^\circ$  otherwise subtract  $180^\circ$ .  $\rightarrow$  IMP ie increase  $\theta$  by  $180^\circ$  Decrease

From arch3!jb4 Fri Jun 5 15:59 EDT 1992

To: jb4, hlwpj!vsh, homxb!sgbali, homxc!lagan, hostar!kpd, hostar!march, mtfmi!mal, mtgzy!erk, mtsol!jcp, mtunp!jim, uhura!lin

Subject: DynaText Demonstration

Status: RO

To: Online Documentation Standards Members  
Re: DynaText Demo in Holmdel

A demonstration of the DynaText product is scheduled for:

Friday June 19, 1992  
Holmdel Room 1E-332  
1:30 to 4:30

The demonstration will be led by a representative from DynaText.

If you plan to attend, please let me know so that I can be sure that the arrangements are adequate. If you have any questions, please give me a call.

Thanks.  
John Baldasare  
arch3!jb4  
908-949-6205

### \* Rectangular to Polar coordinate

$$(x, y) = (r, \theta)$$
$$(3, 4) = (r, \theta)$$

$$9 + 16 = r^2$$

$$r^2 = 25$$

$$r = \pm 5 = 5$$

$$\tan \theta = \frac{4}{3}$$

$$\theta = \tan^{-1} \frac{4}{3} = 53.1$$

(5, 53)

### \* Polar to Rectangular coordinate

$$(4, 60^\circ) \Rightarrow \sin 60^\circ = \frac{y}{4}$$

$$y = 2\sqrt{3}$$

$$\cos 60^\circ = \frac{x}{4}$$

$$x = 2$$

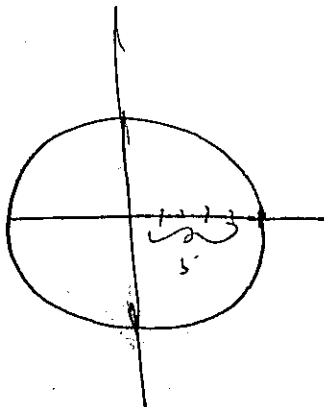
(2, 2√3)

V.V.V.  
IMP

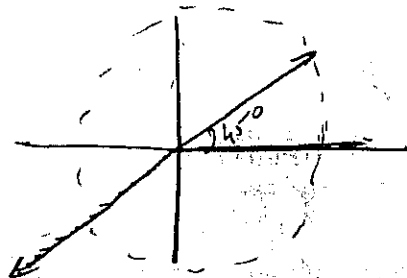
\* IN FINDING THE  $\theta$  MAKE SURE THE ANGLE IS IN THE CORRECT QUADRANT OR HAS THE PROPER SIGN

IMP

\*  $r = 5$



\*  $\theta = 45^\circ$



A polar eq in the form  $r = n \cos \theta$  will give a circle tangent to pole

A polar eq in the form  $r = n \sin \theta$  will give a circle tangent to pole

$r = n \cos \theta$   $\left\{ \begin{array}{l} n \rightarrow \text{is the diameter of the circle} \\ + \text{ive } n \rightarrow \text{to the right} \\ - \text{ive } n \rightarrow \text{to the left} \end{array} \right.$

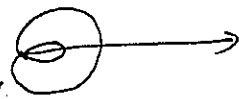
$r = n \sin \theta$   $\left\{ \begin{array}{l} n \rightarrow \text{is the diameter of the circle} \\ + \text{ive } n \rightarrow \text{to the top} \\ - \text{ive } n \rightarrow \text{to the bottom} \end{array} \right.$

$r = a \pm b \cos \theta$  } limacons.

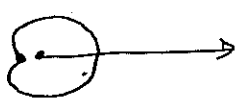
$r = a \pm b \sin \theta$  }

if  $a < b$ , then an extra loop is formed,  
eg  $r = 2 + 3 \cos \theta$

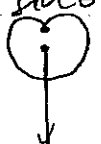
$r = 2 + 3 \sin \theta$



if  $a > b$ , then the indentation moves off pole -  
eg  $r = 3 + 2 \cos \theta$



$r = 3 + 2 \sin \theta$



$r = a \cos n\theta$  } ROSE TYPE  
 $r = a \sin n\theta$

if  $n$  is odd, then have  $n$  petals.

if  $n$  is even, then have  $2n$  petals.

$a$  is the length of the petals.

cosines will be in plus shape.

sines will be in multiplication shape.

A complex # is shown in the form of  $a+bi = z$   
 Real no.  $\rightarrow$  pure imaginary

Complex plane

$a'$

$a$

$b'$

$$r = \sqrt{a^2 + b^2}$$

$$4 + 3i = z$$



$\frac{1}{z}$  is the coefficient of  $\frac{1}{z}$   $\rightarrow$  IMP

To find distance from origin to complex pt.

$$|z| = \sqrt{a^2 + b^2} \quad \text{IMP}$$

$$z_1 = 5 + 2i ; z_2 = 13 - i$$

$$|z_1| = \sqrt{25 + 4} = \sqrt{29} \rightarrow \text{not } \sqrt{21} \text{ because } \frac{1}{z} \text{ is the coefficient}$$

$$|z_2| = \sqrt{8 + 1} = \sqrt{9} = 2$$

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2| = \sqrt{29} \cdot 2 = 2\sqrt{29}$$

$$\frac{|z_1|}{|z_2|} = \frac{|z_1|}{|z_2|} = \frac{\sqrt{29}}{2}$$

$$|3z_1 - 2z_2|$$

$$= |3(2 + 9i) - 2(1 - 2i)|$$

$$= |6 + 27i - 2 + 4i|$$

$$= |-4 + 31i|$$

$$= \sqrt{16 + 961} = \sqrt{977}$$

$z = re^{i\theta}$

### Polar form of a complex system

$$z = r(\cos \theta + i \sin \theta) \rightarrow \text{Polar Form}$$

$$z = \sqrt{2} + i \rightarrow \text{II}^{\text{nd}} \text{ quad.}$$

$$r = 2$$

$$\tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \tan^{-1} \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ = 1 \text{st } \angle$$

$$\text{in II}^{\text{nd}} \text{ quad} = 180 - 30 = 150^\circ$$

$$\therefore 2(\cos 150^\circ + i \sin 150^\circ)$$

$$2(\cos 150^\circ + i \sin 150^\circ)$$

$$2\left(-\frac{\sqrt{3}}{2} + i \frac{1}{2}\right)$$

$$= -\sqrt{3} + i \rightarrow \text{RECTANGULAR FORM}$$

Given polar forms of complex no., to multiply or divide in

$$z_1 \cdot z_2 = r_1 \times r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$$

$$z_1 / z_2 = r_1 / r_2 [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$$

### DE MOIVRE'S THEOREM:

$$z^n = r^n (\cos n\theta + i \sin n\theta)$$

HAVE TO HAVE POLAR FORM TO USE DE MOIVRE'S THEOREM.

To find the  $n^{\text{th}}$  roots of a complex no.

$$\sqrt[n]{r} \left[ \cos \frac{1}{n} (\theta + k \cdot 360^\circ) + i \sin \frac{1}{n} (\theta + k \cdot 360^\circ) \right]$$

$$k = n - 1$$

Substitute in the values of k.

V.V.  
IMI

# LOGARITHMS

log functions are transcendental functions.

$(a^b)^c = a^{bc}$ $\frac{a^b}{a^c} = a^{b-c}$	$a^m \cdot a^n = a^{m+n}$ $4^{1/2} = \sqrt{4} = 2$ $(-8)^{1/3} = \sqrt[3]{-8} = -2$	$\frac{a^m}{a^n} = a^{m-n}$ $16^{1/2} = \sqrt{16} = 4$	$\frac{a^m}{a^n} = \frac{a^m}{a^4} = a^{m-4}$ $36^{-1/2} = \frac{1}{\sqrt{36}} = \frac{1}{6}$
---	---	---	--

CAN TAKE THE ODD ROOT OF A NEGATIVE NUMBER.

For each real no.  $a > 1$ , if  $f(x) = a^x$  defines an exponential funct. whose domain is the set of  $\mathbb{R}$  & Range is  $(0, \infty)$ .

If  $0 < a < 1$ , then  $f(x) = a^x$  is defined as  $f(x) = \frac{1}{a^{-x}}$

If  $a > 1$ , then  $D = \mathbb{R}$  &  $R = (0, \infty)$  ↗ Increasing

If  $0 < a < 1$ , then ↘ Decreasing

If  $a < 0$  then ↔ EXPONENTIAL

$f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x$  ↘ Decreasing

$f(x) = 3 \cdot 2^x$  ↗ IMP RANGE CHANGES when we ADD or SUBTRACT a CONSTANT.

$f(x) = 2^x + 1$  ↗ IMP

$f(x) = -2^x$  ↘

$f(x) = 5^{-x+1} = 5^x \cdot 5$  ↗

$f(x) = 3 - 4^x$  ↘

EXPONENTIAL FUNCS

V.V.V.  
IMP

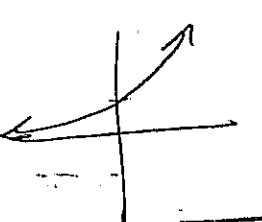


- is being done to re-code inspect it.
- a. What is your definition of "Golden Code"?
- b. How much of your total code is "golden?" Golden Code? c. How do you propose to "reinspect" or
- "walk-through" this Golden Code? d. How much will it cost to do this reinspection?

Reports By: a. 2STP - Mike Kefauver b. 5ESS - Nancy Chapdelaine c.  
4ESS - Jim Foster d. INCP - Jack Rettig

Those persons participating on the conference call in CB and Bedminster should agree among yourselves where to originate the call from. One port per location.

Joanne Jamrok NSD CBU Quality Manager IHC 1G-331 (708)713-5648

- \* base  $e$  is an irrational #
- \* Exponential & log functions are inverse functions
- \* Logs in base 10 are called COMMON LOGS
- \* Logs in base e are called NATURAL LOGS
- \*  $f(x) = e^{7x}$   
 $D = \mathbb{R}^n$   
 $R = (0, \infty)$   
 INC.   
 $y \text{ int} = (0, 1)$   
 $H.A = y = 0$   } V.V.V. IMP
- \*  $e = 2.7182818... \rightarrow$  EULER'S # } IMP
- $e^2 = 7.38$      $e^{2.3} = 10.0$      $e^{-2.3} = 0.1$   
 $e^{-3} = 0.05$      $e^{\sqrt{3}} = 5.5$
- \* Base  $\rightarrow$  to ten-th place.  
 Power  $\rightarrow$  to the 4<sup>th</sup> place.
- \* Log phase is equal to the exponent. It is the inverse exponent fund. } IMP
- \*  $\log_a y = x \Rightarrow a^x = y$  } IMP
- \* Log fund. has a vertical asymptote @  $x=0$  Domain =  $V$ .  
 $\log_4(2x-1)$      $\begin{cases} 2x-1 > 0 \\ x > \frac{1}{2} \end{cases}$  } V.V.V. IMP } V.V.V. IMP } IMP

$\log(x) = y$   
 $x > 0$   
 $x > 1$   
 $x > 10$



$\log_2 8 = 3$   
 $\log_2 4 = 2$   
 $\log_2 2 = 1$   
 $\log_2 1 = 0$

$\log_2 8 = 3$   
 $\log_2 4 = 2$   
 $\log_2 2 = 1$   
 $\log_2 1 = 0$

WHOLE # RACE → Imp  
 $\log y = n$   
 $D = (0, \infty)$   
 $x \text{ int} = (1, 0)$   
 $V.A. = n = 0$   
 $R = f' b$   
INC  

V.V.V.  
IMP

\* A DECIMAL BASE → Imp  
 $D = (0, \infty)$   
 $R = f' b$   
 $(1, 0)$   
 $V.A. = n = 0$   
dec = base of 1/9

Common logs have a base of 10  
 $\log_{10} 496 = 2.6955$

antilog →  $10^{2.6955} = 496$

Natural logs have a base of e

$\ln 38 = n$   
 $e^n = 38$   
 $\ln 38 = 3.6376$   
 $\{ e^n = 38, e^{3.6376} = 38 \}$  Imp

V.V.V.  
IMP

log phrases are exponents.

1.  $\log_a xy = \log_a x + \log_a y$

2.  $\log_a \frac{x}{y} = \log_a x - \log_a y$

3.  $\log_a x^y = y \log_a x$

IMP

From homxb!sgbali Mon Apr 13 16:58 EDT 1992 From: homxb!sgbali (Shri G Bali +1 908 949 0281) To: hlwpj!jolang Subject: printing Status: RO

>From attlattmail!dauer Mon Apr 13 13:16:40 GMT 1992 Message-Version: 2 >To: attbl!homxb!sgbali Date: Mon Apr 13 08:19:25 CDT 1992 From: attmail!dauer (David Louis Auer) Received: from dauer by attmail; Mon Apr 13 13:15:43 GMT 1992 MTS-Message-ID: <dauer1041315490> UA-Content-ID: <ATT-2.04-dauer-0000000000-222> End-of-Header: EMail-Version: 2 Phone: 708-224-6844 Fax-Phone: +1 708 224 4583 Subject: AMD QIT CONF. CALL UA-Message-ID: <ATT-2.04-dauer-0000000000-222> To: attmail!attbl!wrddol!kmt To: attmail!attbl!ihlpf!rrush To: attmail!attbl!homxb!kpd To: attmail!attbl!homxb!sgbali To: attmail!lvick (Lowell Vick) To: attmail!dauer (David Louis Auer) To: attmail!hlwatson (Harold L Watson) To: attmail!bnroy (Bentley E Roy) To: attmail!vathompson (Valerie Ann Thompson) To: attmail!attbl!swsigdiv!wdr To: attmail!cfrost (Clayton W Frost) To: attmail!nesac2!bds To: attmail!butz (Jerry D Butz) End-of-Protocol: Content-Type: Text Content-Length: 387 Status: RO

The Tuesday April 14 AMD QIT meeting has been rescheduled due to a managers conference in Bedminster. The rescheduled conference call will be held on 4/20 (Monday) from 2-4 P.M. EST to review AMD trial status. This call should not take the full 2 hours. If anyone is in Denver we will be in conference room G1.

The alliance meet me bridge # is 0-700-452-2634. Jerry Boudreau/Dave Auer

\* CANNOT TAKE THE LOG OF A NEGATIVE NUMBER. } IMP

\* ~~log~~ mantissa wherever necessary.

\* CHANGE OF BASE RULE

a).  $\log_a b = \log b / \log a$   
1).  $\log_3 7 = \log 7 / \log 3 = \approx 1.7712$

$\log_3 7 = \frac{\log 7}{\log 3} \rightarrow n = \frac{\log 7}{\log 3}$

IMP IMP

\* USE COMMON OR NATURAL LOGS APPROPRIATELY WHEREVER NECESSARY.

\* MANTISSA → # greater than 0 & less than one

CHARACTERISTIC → whole #

\* Mantissa is always POSITIVE.

$10^{0.5775+2} = 10^{0.5775} \times 10^2$   
 $= 3.78 \times 10^2 = \underline{\underline{378}}$

IMP

## SEQUENCES

Sequence - a pattern of numbers where the domain is  $\mathbb{N}$  and the range is generated from the given sequence formula.

$a_n \rightarrow n$  is the no. of terms.

$$a_n = \{a_n\} = \left\{ f(n) = \frac{1}{n} \right\}$$

There are Arithmetic & Geometric Sequences.

### ARITHMETIC SEQUENCE OR PROGRESSION

If each term after the 1st term differs from the preceding terms by a fixed number.

$$a_n = a_1 + (n-1)d \quad \} \text{IMP}$$

$d$  = Common Difference { 2nd term - 1st term }

GEOMETRIC PROGRESSION  $\rightarrow$  where each term after the 1st term is formed by multiplying the preceding term by a fixed no.

$$a_n = a_1 \cdot r^{n-1} \quad \} \rightarrow \text{IMP}$$

$r$  = Common ratio { the ratio of the 2nd term over the 1st term }

Sum of an infinite sequence is called a series.

The compact form uses sigma notation.

$$\sum_{k=1}^n (2k+3) \quad \} \underline{n} \rightarrow \text{INDEX OF SUMMATION}$$

$\rightarrow$  STARTING INTEGER

$\sum_{k=1}^6 (2k-4) \rightarrow$  Compact form of the series (sum)

$[2(1)-4] + [2(2)-4] + [2(3)-4] + \dots + [2(6)-4] \rightarrow$  Expanded form

$-1 + 2 + 5 + 8 + 11 + 14 \rightarrow$  Series form

$39 \rightarrow$  sum

SIGMA NOTATION ALWAYS REPRESENTS SUMMATION.

### PROPERTIES OF SUMMATION

1) Constant Property  $\rightarrow \sum_{k=1}^n c = \boxed{nc}$

2) Homogenous Property  $\rightarrow \sum_{k=1}^n (ca_k) = \boxed{c \sum_{k=1}^n a_k}$

3) Additive Property  $\rightarrow$

$$\sum_{k=1}^n (a_k + b_k) = \boxed{\sum_{k=1}^n a_k + \sum_{k=1}^n b_k}$$

4) Sum of successive Integers  $\rightarrow$

$$\sum_{k=1}^n k = \boxed{\frac{n(n+1)}{2}}$$

5) Sum of successive squares  $\rightarrow$

$$\sum_{k=1}^n k^2 = \boxed{\frac{n(n+1)(2n+1)}{6}}$$

I  
M  
P

TO USE THE PROPERTIES OF SUMMATION, THE  $k$  VALUE SHOULD ALWAYS START FROM ONE  $\rightarrow$  MOST IMP.

Sum of A.P.  $\rightarrow S_n = \frac{n}{2} [2a_1 + (n-1)d]$  IMP.  
 $S_n = \frac{n}{2} (a_1 + a_n)$

SUM OF GP  $\rightarrow S_n = \frac{a_1(1-r^n)}{1-r}$  (where  $r \neq 1$ ) IMP

SUM OF INFINITE SERIES  $\rightarrow S_n = \frac{a_1}{1-r}$  (where  $|r| < 1$ ) IMP

$0.\bar{3} = 0.3 + 0.03 + 0.003 + \dots$

$a_1 = 0.3$

$r = 0.1$  or  $1/10$

$S_n = \frac{0.3}{1 - \frac{1}{10}} = \frac{3}{9} = \frac{1}{3}$

$10n = 3.\bar{3}$   
 $n = 0.\bar{3}$   
 $10n = 3.0$

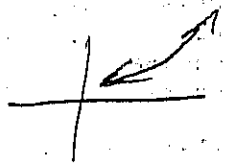
$n = \frac{3}{9} = \frac{1}{3}$

$\sum_{k=1}^{\infty} 2^k$

$2 + 4 + 8 + 16 + 32 + \dots$

$r = 2 \rightarrow$  ratio is greater than one

DIVERGENT SUM



FACTORIAL NOTATION

$n!$  is defined for all non-negative integers.

$0! = 1$   $\rightarrow$  IMP

$n! = n(n-1)(n-2)(n-3) \dots$

COMBINATION NOTATION

$k$  &  $n$  are IP such that  $0 \leq k \leq n$

then  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$  IMP

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

$$\frac{n}{2} [a_1 + a_n]$$

$$a_n = a_1 \cdot r^{n-1}$$

$$S_n = \frac{a_1(1-r^n)}{1-r} \rightarrow r \neq 1$$

$$S_n = \frac{a_1}{1-r} \text{ where } |r| < 1$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$0! = 1$$

Constant  $\sum_{k=1}^n c = nc$

Homogenous  $\sum_{k=1}^n c \cdot r^k = c \sum_{k=1}^n r^k$

Additive  $\sum_{k=1}^n (a_k + b_k) =$

$$\sum_{k=1}^n a_k + \sum_{k=1}^n b_k$$

Success Int  $\rightarrow \sum_{k=1}^n k$

$$\frac{n(n+1)}{2}$$

Success Square  $= \sum_{k=1}^n k^2$

$$\frac{n(n+1)(2n+1)}{6}$$

BINOMIAL EXPANSION  $\rightarrow$

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \binom{n}{3} a^{n-3} b^3 + \dots + \binom{n}{n} b^n$$

$$\sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

To find a particular term of the binomial expansion

(Power) ( )  $\xrightarrow{\text{diff. between power } \& (n-1)}$  ( )  $\rightarrow$  V.V. I.T.P

$(2n-y)^8 \rightarrow$  6th term

$\binom{8}{6-1} (2n)^3 (-y)^5 \Rightarrow -448n^3 y^5$

From arch3!jb4 Thu Jun 4 10:10 EDT 1992

To: alfalfa!cec, arch1!lhh, arch2!dnn, bct, deronda, jb4, lbb,  
attmail!ckhoffman, attmail!ebron, attmail!joanneb, attmail!laffey,  
attmail!mcarey, attmail!middaugh, attmail!trohalides, attmail!wfish,  
attme!alkab, drddol!mary, druks!ouray, druks!wolpert, fjtlc!allenb,  
homxb!sgbali, homxc!lagan, hoqax!acj, hostar!kpd, ihlpw!straka,  
ihlpy!napatch, indycic!djr, indycic!gil, mtdcb!pdn!j.new,  
mtdcb!pdn!k.fitzgerald, mtfme!lmk, mtfme!tml, mtfmi!mal, mtgzy!erk,  
mtunp!jim, mtunp!nee, mtuxo!llhl, nwwpa!wjb, techman!trb, uhura!lin,  
uhura!nam, violin!gpp, wrddo!cnile, wrddo!lgj

Subject: DynaText Demo  
Status: RO

To: Online Documentation Standards Members  
Re: DynaText Demo in Holmdel

A demonstration of the DynaText product is  
scheduled for:

Friday June 19, 1992  
Holmdel Room 1E-332  
1:30 to 4:30

The demonstration will be led by a representative  
from DynaText.

If you plan to attend, please let me know so that  
I can be sure that the arrangements are adequate.  
If you have any questions, please give me a call.

Thanks.  
John Baldasare  
arch3!jb4  
908-949-6205

\*  $(n + \sqrt{a})^{12}$ , middle term  
for 0 to 12, the middle term is

$$\binom{12}{6} (n)^6 (\sqrt{a})^6$$

$$= 924 n^6 a^3$$

\* NO MIDDLE TERM FOR ~~ADD~~ POW  
JMB  
3



CONICS

Circle - the set of points a fixed distance from a given point (center)

standard form of a circle at origin =  $x^2 + y^2 = r^2$

general form of a circle off the origin =  $(x-h)^2 + (y-k)^2 = r^2$

Center =  $(h, k)$  & Radius =  $r$

In a circle the two quadratic variables are added & their numerical coefficients are the same.

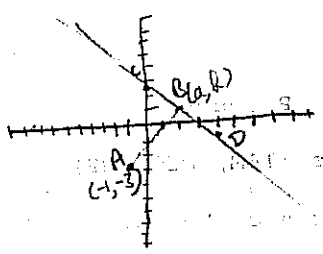
distance formula:  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

midpoint formula =  $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

slope formula:  $\frac{y_2 - y_1}{x_2 - x_1}$  or  $\frac{\text{rise}}{\text{run}}$  or  $\frac{\sqrt{\text{what's under y}}}{\sqrt{\text{what's under x}}}$  } IMP

C =  $(-1, -3)$  & tangent to  $3x + 4y = 10$

$3x + 4y = 10$   
 $= y = \frac{-3x + 10}{4}$   
 $(x+1)^2 + (y+3)^2 = 25$



slope of AB =  $\frac{-3 - b}{-1 - a}$

slope of CD =  $\frac{-3}{4}$

$\Rightarrow \frac{-3 - b}{-1 - a} = \frac{4}{3}$

opposite reciprocal → IMP

$-9 - 3b = -4 - 4a$   
 $4a - 3b = 5$

$\Rightarrow \begin{cases} 4a - 3b = 5 \\ 3a + 4b = 10 \end{cases} \Rightarrow$

$12a - 9b = 15$   
 $12a + 16b = 40$   
 $-25b = -25$   
 $b = 1$

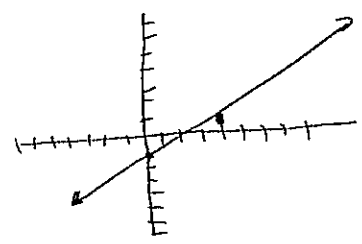
$4a - 3 = 5$   
 $a = 2$

$\therefore \sqrt{(2+1)^2 + (1+3)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\therefore \text{circle} = (x+1)^2 + (y+3)^2 = 25$

contains pt  $(-3, 7)$  &  $(5, 5)$  & center on line  $x - 4y = 1$

$x - 4y = 1$   
 $y = \frac{x}{4} - \frac{1}{4}$



$(h-k)^2 + (y-k)^2 = r^2$   
 $(-3-h)^2 + (7-k)^2 = r^2$   
 $(5-h)^2 + (5-k)^2 = r^2$   
 $9 - 6h + h^2 + 49 - 14k + k^2 = r^2$   
 $-25 = 10h + h^2 + 25 = 10k + k^2 = r^2$   
 $-16 + 4h + 24 - 4k = 0$

$\Rightarrow \begin{cases} 4h - 4k = -8 \\ 4(h-k) = -8 \\ h - k = -2 \end{cases}$

$(h+3)^2 + (y+1)^2 = 10$   
 $\Rightarrow \begin{cases} h - k = -2 \\ h + 4k = 1 \\ 3k = -3 \\ k = -1 \\ h + 1 = -2 \\ h = -3 \end{cases} \Rightarrow (-1, -3)$   
 $\sqrt{64 + 36} = \sqrt{100} = 10$

From arch3!oca Fri Jun 5 17:37 EDT 1992  
 To: arch3!oca  
 Subject: competitive briefing next week  
 Status: RO

Reminder: The AT&T Architecture Area will be reviewing the technical and architectural directions of AT&T's competitors.

This briefing will be repeated in four locations from 9:00am to 12Noon as follows:

- June 9 - Holmdel 5E-201 (mini-auditorium)
- June 10 - Bedminster Auditorium
- June 11 - Dayton (NCR Sugarcamp)
- June 12 - Indian Hill Auditorium

No reservations are required, but only AT&T employees may attend.

The briefing will be divided into two parts. The agenda is:

Part 1 - 9:00am to 10:15 - Telecommunications Products & Services

Subject	Example Vendors
Networking Products Vendors	Alcatel, Northern Telecom
Inter-exchange carriers	MCI, Sprint
RHCs / AAVs	US West, MFS
Global Carriers & PTOs	BT, C&W, Sprintnet
Wireless	Motorola, McCaw
Video Telephony	PictureTel

Part 2 - 10:30 to 11:30 - Computer Products

- Major Domestic Competitors (IBM, DEC, HP)
- The Japanese Computer Industry
- Application and Networking Architectures (SAA/SNA, OCCA, ...)
- Fault-Tolerant Systems
- Industry Directions, Trends and Strategies

- \* ELLIPSE → set of all pts. where the sum of the distances from 2 fixed pts (foci) is constant.
- \* Gen eqn →  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
- \* center = (h, k)
- \* MAJOR AXIS → ABOVE THE GREATER DENOMINATOR.
- \* Endpoints of the major axis =  $\sqrt{\text{GREATER DENOMINATOR}}$
- \* Length of the major axis is = 2 X ENDPOINT.
- \* FOCUS = c ⇒  $c^2 = a^2 - b^2$  IMP  
     ↳ BIGGER DENOMINATOR
- \*  $a^2$  is always the bigger denominator.
- \* THE FOCI ALWAYS LIE ON THE MAJOR AXIS.
- \* IN ELLIPSE BOTH THE QUADRATIC VARIABLES ARE ADDED
- \* IN ELLIPSE THE NUMERICAL COEFFICIENTS OF THE QUADRATIC VARIABLES ARE DIFFERENT
- \* FOCAL ORD → Segment ⊥ to the major axis & passes through the foci (mid pt. of the)
- \*  $F.C = \frac{2b^2}{a}$  IMP

V.V.V.  
IMP

LEARN THE "COMPLETING THE SQUARE" METHOD.

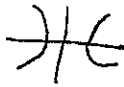
$a$  → is the distance from the center to the endpt of the major axis.


$c$  → is the distance from the center to the foci.

HYPERBOLAS → set of all pts where the difference between the distance of two fixed pts. (foci) is constant.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad \text{or} \quad \frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

$h$  always lies under the five variable. → IMP

+ve  $x^2$  term will stretch the graph left & right. 

-ve  $x^2$  term will stretch the graph up & down. 

TRANSVERSE AXIS → is always with the five denominator or  $a^2$ .

Center  $(h, k)$

Focus  $c^2 = a^2 + b^2$   
 ↳ +ve denom.

IN HYPERBOLA BOTH THE QUADRATIC VARIABLES ARE CONTRACTED.

The numerical coefficients do not matter

Focus cord =  $\frac{2b^2}{a}$

EQUATION OF A LINE OR ASYMPTOTE =  $y = \pm mx$  } IMP  
 ↳ SLOPE

PARABOLAS → a set of pts. formed when the distance from a point on the curve to a fixed pt (focus) is the same as the distance from the pt (P) to a fixed line (directrix).

$(x-h)^2 = \pm 4c(y-k)$  → opens up or down

$(y-k)^2 = \pm 4c(x-h)$  → opens left or right.

IMP

$c$  is the distance from the vertex (center) to the focus then to the directrix.

Focus cord =  $|4c|$ ; Focus =  $c$

Equation must be solved for the QUADRATIC VARIABLE with a coefficient of +ive or -ive

AXIS OF SYMMETRY IS ⊥ TO THE DIRECTRIX.

From pav Wed Jun 3 14:51 EDT 1992

From: hoqax!pav (Paul Vasilopoulos +1 908 949 0278)

To: attmail!crysel (Cheryl K Crysel), attmail!swientek (Christine A Swientek),  
attmail!dmckay (David G Mckay),  
attmail!dstevenson (Donald L Stevenson, Jr),  
attmail!dethomas (Dwight E Thomas, Jr), attmail!gmayer (Gary A Mayer),  
attmail!gnunnally (Glyn D Nunnally), attmail!gmharris (Gloria M Harris),  
attmail!jneal (James A Neal), mvgpk!mvjc8 (James M Crowley),  
attmail!jjfinnegan (John J Finnegan),  
attmail!kdunnahoo (Lillian K Dunnahoo), attmail!luciej (Lucie M Johnson),  
attmail!mewalker (Marsha E Walker), attmail!msell (Mary L Sell),  
attmail!nmcgrath (Nancy L Mcgrath), attmail!pfennell (Patricia E Fennell),  
homxb!pav (Paul Vasilopoulos), attmail!kaissling (Ronald C Kaissling),  
attmail!rjrichter (Robert J Richter), attmail!rsayers (Ronald S Ayers),  
homxb!sgbali (Shri G Bali), attmail!tmallory (Teri M McMahon),  
hlwpj!vsh (Victoria S Hierung), attmail!wnoles (Wilbur R Noles),  
so043b!wtb (William T Barr), attmail!wbradford (Willene B Bradford)

Subject: Motorola

Status: RO

The Motorola benchmarking visit has been registered with Continental, Northwest, Delta, and United. America West does not register AT&T meetings (at least according to American Express Travel).

The name given to the meeting was Motorola Benchmarking, and Shri Bali is the point of contact for the airlines.

Paul

TRANSLATION OF AXIS → created a new coordinate system where (h, k) becomes the new origin system called the  $\bar{x}\bar{y}$  system.

$$x^2 = 8y \Rightarrow \bar{x}^2 = 8\bar{y}$$

$$\begin{cases} \bar{x} = x - h \\ \bar{y} = y - k \end{cases} \text{ } \underline{\underline{\text{IMP}}}$$

Conic sect → any curve obtained from the intersection of double napped cones & a plane.

\* Double napped cones are Right circular cones.

\* Plane parallel to the base of a cone gives a circle.

\* Plane intersecting a cone at an angle gives an ellipse.

\* Plane  $\perp$  to axes & passes through both ends gives a hyperbola.

\* Plane cuts through base & cone gives parabola.

### APOLLONIUS

Defn. Conics: → determined by a given pt. F (focus), a given line d (directrix) not containing the focus, and a positive number e (eccentricity). The conic contains a point P iff  $\left| \frac{FP}{PD} \right| = e$ , where D is foot of  $\perp$  from P to the line d.

$$e = \left| \frac{FP}{DP} \right| \text{ } \underline{\underline{\text{IMP}}}$$

ECCENTRICITY MUST BE A POSITIVE NUMBER.

If  $e = 1$ , it gives a parabola.

If  $0 < e < 1$ , it gives an ellipse.

If  $e > 1$ , it gives a hyperbola.

If  $e = 0$ , it gives a circle.

IMP

ECCENTRICITY =  $e = \frac{c}{a}$

Directrix =  $x = \pm \frac{a^2}{c}$

IMP

DEGENERATE CONICS → are formed when the plane intersecting a cone and the intersection is a POINT. If the planes intersect the cone & so get two intersecting lines.

Rotation of axis, origin remains same & axes rotates counter clock with an  $\theta$  of  $\theta$ .

$$\bar{x} = x \cos \theta + y \sin \theta$$

$$\bar{y} = y \cos \theta - x \sin \theta$$

$$x = \bar{x} \cos \theta - \bar{y} \sin \theta$$

$$y = \bar{y} \cos \theta + \bar{x} \sin \theta$$

MOST IMP

GENERAL FORM OF A QUADRATIC

a)  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

b)  $A\bar{x}^2 + C\bar{y}^2 + D\bar{x} + E\bar{y} + F = 0$

↳ -Bxy is skipped & D $\bar{x}$  is positive.

IMP

To find the  $\theta$  of rotation,

→  $\cot 2\theta = \frac{A-C}{B}$  } LOOK AT THE TABLE IN THE BOOK.

RECTANGULAR HYPERBOLA →  $xy = 4$  IMP

From pav Wed Jun 3 15:16 EDT 1992

From: hoqax!pav (Paul Vasilopoulos +1 908 949 0278)

To: hlwpj!diane, hlwpj!jolang, hlwpj!vsh, homxb!fine, homxb!jtupino, homxb!khn,  
homxb!kpd, homxb!march, homxb!norris, homxb!sgbali, homxc!darla, hoqax!pav,  
homxb!tlr (Thomas L Russell, Jr)

Subject: login

Status: RO

As part of my move to Hosein Fallah's department, my e-mail address has changed to hoqax!pav. For now, my mail is being forwarded from homxb, but this will shortly go away.

Paul

### LINEAR SYS. & EQUATIONS

The graphs of two linear equations intersect at one point; we say the system is CONSISTENT.

The graphs of two linear equations intersect at no point; we say that the system is INCONSISTENT.

The graphs of two linear equations coincide so that the intersection is every point of each line; we say the system is DEPENDENT.

Graph of linear & linear  $\rightarrow$  one or none or all.

Graph of lin & quad  $\rightarrow$  none, one or two.

Graph of quad & quad  $\rightarrow$  none, one, two, three, or four solutions.

quadratic & linear equations can be solved algebraically using -

- a. SUBSTITUTION
  - b. ELIMINATION
- } IMP

SUBSTITUTION IS used only if ONE OF THE VARIABLES has the NUMERICAL COEFFICIENT OF ONE.