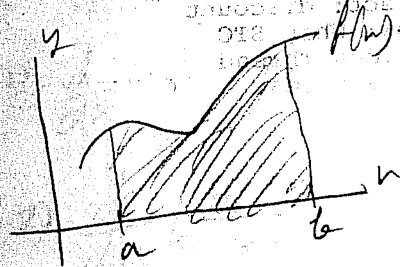
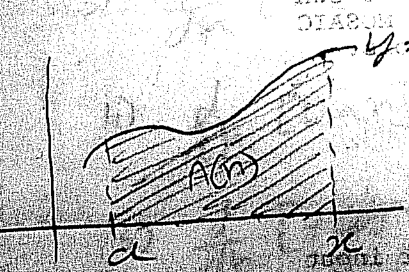
Utsav Bali
Calculus Review
1991- 1993

5. INTEGRATION

INTEGRATION



* Given a function f that is continuous and non negative on an interval [a, b], find the area between the graph of f & the interval [a, b] on the x-axis.



+ Area of this continuous curve on the interval [~~a(fixed)~~, x(va ... [a (fixed, x (variable)] is dependent upon x or Area is a function of x.

~~A(x)~~ NEWTON & LIEBNIZ showed that $A'(x) = f(x)$

or $\int f(x) = A(x)$

+ ~~Thus~~ if we are given a function f(x) on interval [a, x], then that function is the derivative of its area & so if we can reverse the process or take the antiderivative of the ~~function~~ function, we shall get the eq. for the area which can be evaluated by plugging in x in the function of Area i.e. Area at x int. [a, x] or [a, b] = A(x) or A(b)

+ Area under the graph of $f(x) = x^2$ on [0, 1]

∴ antidifferential of $f(x) = \frac{1}{3}x^3 + C$

where (C) is a CONSTANT & ~~anything multiplied or added to a constant is a constant.~~

+ However, if we are given $A(0) = 0$, then we can find out the value of c

$0 = 0 + C$

∴ $C = 0$

∴ the equation for the area is

$A(x) = \frac{x^3}{3}$

QUEST Contact: J. Maranzano, G. Pasternack, R. Mendez
Results: Good discussion. Interest in OOT (but using their methodology), estimation service, arch. consulting, & process eng. Discussed deal for volume contract: discount SABLIME for 3 yr. contract. They want to know: What STC products don't use SABLIME, NMAKE, & what Golden Thread metrics QUEST uses to manage our products & services. Do we walk the talk?

Item 29:

Date: 10/30/92
Customer: FSAT Product Mgmt Team
Event: Present MOSAIC/SDE
Customer Contact: Dave Weiner
QUEST Contact: J. Maranzano, G. Pasternack
Results: Interest in OOT, SDE tools. They've created their own Process Asset Library for FSAT. Duplication of MOSAIC work. Our main entry will be thru SDE & SDE tools.

END REPORTS

If you have any items for the Customer Events Calendar, send your input to George Grant (HO 1D-406; x6562; hogpa!gsg1; Fax x6868), or directly to hogpa!custcal. The next edition will appear on December 2nd. Please send in any customer related item(s) for the next release by November 30th.

ANTIDERIVATIVES: THE INDEFINITE INTEGRAL

* A function F is called an ANTIDERIVATIVE of a function f if the derivative of F is f.

* If F(x) is any antiderivative of f(x), then for any value of C, the function F(x) + C is also an antiderivative of f(x); moreover on any interval, every antiderivative of f(x) is expressible in the form of F(x) plus constant.

* The process of finding antiderivatives is called ANTIDIFFERENTIATION or INTEGRATION. It is denoted by the symbol '∫' followed by the eq. & (dx). ——— (1)

$$\int f(x)\,dx = F(x) + C$$

* Symbol '∫' is the Integral sign, & this is an INDEFIN[ITE] INTEGRAL of f(x) as the right side of 1 is not a definite function, but rather a whole set of possible functions; the constant C is called the CONSTANT O[F] INTEGRATION.

* The symbol (dx) as in ∫[ ]dx serves to IDENTIFY THE INDEPENDENT VARIABLE.

DIFFERENTIATION FORMULAS

1) $\frac{d}{dx}[x] = 1$

2) $\frac{d}{dx}\left[\frac{x^{r+1}}{r+1}\right] = x^r \ (r \neq 1)$

3) $\frac{d}{dx}[x^N] = N x^{N-1}$

4) $\frac{d}{dx}[\sin x] = \cos x$

5) $\frac{d}{dx}[+\cos x] = -\sin x$

6) $\frac{d}{dx}[\tan x] = \sec^2 x$

7) $\frac{d}{dx}[\cot x] = -\csc^2 x$

8) $\frac{d}{dx}[\sec x] = \sec x \tan x$

9) $\frac{d}{dx}[\csc x] = -\csc x \cot x$

INTEGRATION FORMULAS

1) $\int 1\, dx = x + C$

2) $\int x^r dx = \frac{x^{r+1}}{r+1} + C \ (r \neq 1)$

3) $\int N x^{N-1} dx = N \int x^{N-1} dx = \frac{N \cdot x}{(N-1)+1} + C$

4) $\int \cos x\, dx = \sin x + C$

5) $\int -\sin x\, dx = \cos x + C$

6) $\int \sec^2 x\, dx = \tan x + C$

7) $\int -\csc^2 x = \cot x + C$

8) $\int \sec x \tan x\, dx = \sec x + C$

9) $\int -\csc x \cot x\, dx = \csc x + C$

10) $\int \tan x$ = Integ. by parts.

11) $\int \cot x$ = Integ. by parts.

12) $\int \sec x = \int \csc x$ = Integ. by parts.

$$\frac{d}{dx}\left[\int f(x)dx\right] = f(x)$$

a) A constant factor can be moved through an integral sign — ONLY A CONSTANT (NUMBER) & nothing else.

$$\int c f(x)\, dx = c \int f(x)\, dx$$

b) An antiderivative of a sum is the sum of the antiderivatives, i.e. —

$$\int [f(x) + g(x)]\, dx = \int f(x)\, dx + \int g(x)\, dx$$

also

$$\int [f(x) - g(x)]\, dx = \int f(x)\, dx - \int g(x)\, dx$$

* $\int x^{-1}\,dx = \int \frac{1}{x}\,dx$ [UNSOLVABLE AT THE MOMENT]

* $\int [f(x)\cdot g(x)]\,dx \neq \int f(x)\,dx \cdot \int g(x)\,dx$ — IMP

We can break up only sum or difference & not product or quotient.

* We also cannot integrate unless the function has only one kind of variable.

eg $\int x^2 + y + 1$ = No. $\int x^2 + x + 1$ = yes

* $\int f''(x) = f'(x) + c$ $\qquad \frac{d}{dx}[f(x)] = f'(x)$

$\int f'(x) = f(x) + c$ $\qquad \frac{d}{dx}[f'(x)] = f''(x)$

INTEGRATION BY SUBSTITUTION

* $\int \left[f(u)\frac{du}{dx}\right] dx = \int f(u)\,du$

* Let $\frac{d}{du}[F(u)] = f(u)$ or $\int f(u)\,du = F(u) + c$

* If u IS A DIFFERENTIABLE FUNCTION OF x, then

$$\frac{d}{dx}[F(u)] = \frac{d}{dx}[F(u)]\cdot\frac{du}{dx} = f(u)\frac{du}{dx}$$

or

$$\int \left[f(u)\frac{du}{dx}\right] dx = F(u) + c$$

IMP

* TO INTEGRATE BY SUBSTITUTION:

STEP 1: Make a choice for u, say $u = g(x)$

STEP 2: Compute $\frac{du}{dx} = g'(x)$

STEP 3: Make the substitution $u = g(x)$ & get

$$\frac{d[g(x)]}{dx} = g'(x)$$

or $\boxed{du = g'(x)\,dx}$ → IMP

At this stage, the entire integral should be in terms of u, no x's should remain. If not so, try a different substitution for u or try replacing other values w/ u if possible. YOU ARE ALLOWED TO MANIPULATE THE EQUATION BY PUTTING A CONSTANT OUTSIDE THE INTEGRATION SIGN BUT YOU CANNOT PUT A VARIABLE OUTSIDE THE SIGN

IMP

STEP 4: Evaluate the resulting integral.

STEP 5: Replace u by $g(x)$, so the final answer is in terms of x.

SIGMA NOTATION

* $1^2+2^2+3^2+4+5 \rightarrow$ EXPANDED FORM

$\sum_{k=1}^{5} k^2 \rightarrow$ SHORT FORM

$5 \rightarrow$ UPPER INDEX

$k=1 \rightarrow$ LOWER INDEX

* NUMBER OF ITEMS = (UPPER INDEX − LOWER INDEX) +1 } MOST IMP

* $\sum_{K=1}^{5} 2K = 2+4+6+8+10$

$\sum_{K=0}^{4} 2K+2 = 2+4+6+8+10$

$\sum_{K=2}^{6} 2K-2 = 2+4+6+8+10$

* $\sum_{K=3}^{7} 5^{K-2}$ change K=3 to K=0

We take J such that J=0 w/ respect to K

$\therefore J = K-3$ on substitution −

$\therefore K = J+3$

$\sum_{J=0}^{(7-3)} 5^{(J+3)-2} = \sum_{J=0}^{4} 5^{J+1}$

If we want K=5, then

$J = K-2$

$\therefore K = J+2$

$\sum_{J=5}^{9} 5^{(J+2)-2}$

$= \sum_{J=5}^{9} 5^{J}$

IMP

1) $\sum_{K=1}^{N}(a_{K}+b_{K}) = \sum_{K=1}^{N} a_{K} + \sum_{K=1}^{N} b_{K}$

2) $\sum_{K=1}^{N}(a_{K}-b_{K}) = \sum_{K=1}^{N} a_{K} - \sum_{K=1}^{N} b_{K}$

3) $\sum_{K=1}^{N} C a_{K} = C \sum_{K=1}^{N} a_{K}$

4) $\sum_{K=1}^{N} K = 1+2+3+4+\dots+N = \boxed{\frac{N(N+1)}{2}}$ IMP

IMP 5) $\sum_{K=1}^{N} K^{2} = 1^{2}+2^{2}+3^{2}+4^{2}+\dots N^{2} = \boxed{\frac{N(N+1)(2N+1)}{6}}$

6) $\sum_{K=1}^{N} K^{3} = 1^{3}+2^{3}+3^{3}+\dots+N^{3} = \boxed{\left[\frac{N(N+1)}{2}\right]^{2}}$ IMP

EVALUATE $\sum_{K=1}^{N} (3+K)^{2}$

N IS A CONSTANT & NOT A VARIABLE

$$\sum_{K=1}^{N} (9+6K+K^{2})$$

$$= \sum_{K=1}^{N} 9 + 6\sum_{K=1}^{N} K + \sum_{K=1}^{N} K^{2}$$

$$= 9N + \cancel{6}^{3}\left[\frac{N(N+1)}{\cancel{2}}\right] + \frac{N(N+1)(2N+1)}{6}$$

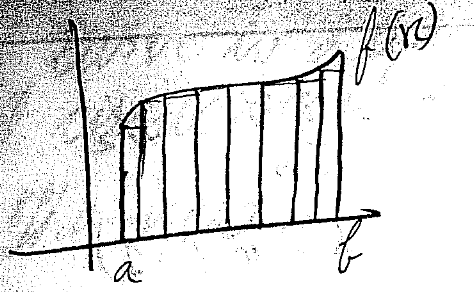
$\sum_{K=1}^{N} C$ (constant) $= C \cdot N$ IMP

$$= 9N + 3N^{2} + 3N + \frac{2N^{3}+N^{2}+2N^{2}+N}{6}$$

$$= \frac{54N+18N^{2}+18N+2N^{3}+N^{2}+2N^{2}+N}{6}$$

$$= \frac{2N^{3}+21N^{2}+73N}{6}$$

5.5 AREAS AS LIMITS

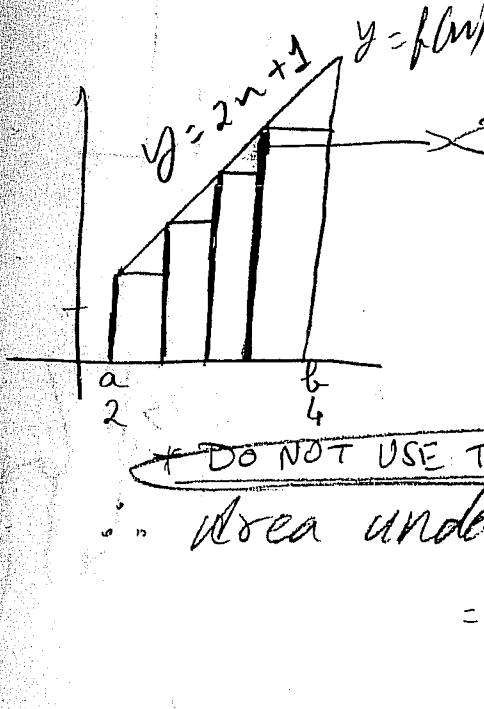


* FUNCTION HAS TO BE ABOVE X-AXIS

R = Area of each rectangle . & so

Area under the curve = $\sum_{N=a}^{b} R_N$ (sum of the areas of all the rectangles from a to b)

Area = $\lim_{N\to\infty} \sum_{N=a}^{b} R_N$ (more accurate)



→ INSCRIBED RECTANGLES [2, 4]

$$\boxed{\Delta x = \frac{b-a}{N}} \rightarrow \text{IMP}$$

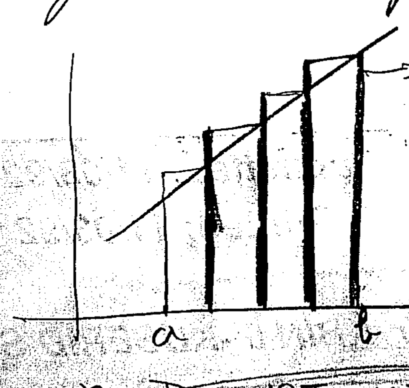
$N = 4$ $\Delta x = \frac{4-2}{4} = \frac{1}{2}$

* DO NOT USE THE 'b' VALUE → VERY IMP

∴ Area under the curve $y = f(x)$

$$= \boxed{\sum_{i=1}^{N} f(C_i) \times \Delta x}$$

* To get better approximations we make 'N' larger

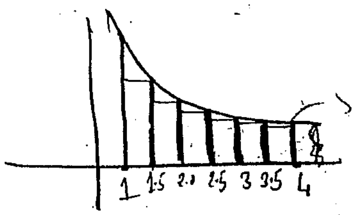


→ CIRCUMSCRIBED RECTANGLES

IMP We use 'C_i' when it is the Mi[n] & D_i when it is the Max.

* DO NOT USE THE 'a' VALUE

* $f(x) = \frac{1}{x}$ [1, 4] , N = 6 $\Delta x = \frac{4-1}{6} = 1/2$



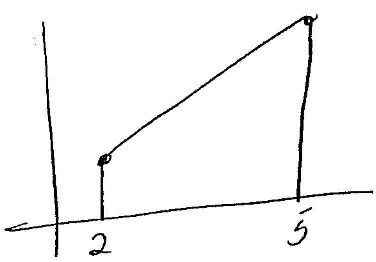
Use Inscribe. (Do not use C = 1)

SGBALI

$C_1 = 1.5$
$C_2 = 2.0$
$C_3 = 2.5$
$C_4 = 3.0$
$C_5 = 3.5$
$C_6 = 4.0$

$\sum_{i=1}^{6} f(C_i) \times \Delta x \approx$ Area.

* $y = 2x - 3$ [2, 5] , N = 4



Circumscribed = 14.25
Inscribed = 9.75

Area = $9.75 < A < 14.25$

If N = 6 , then $10.5 < A < 13.5$
If N = 6 , then $11.25 < A < 12.75$

* As $N \to \infty$, the area becomes more exact. If the curve is a line then the average of inscribed and circumscribed area is the exact number.

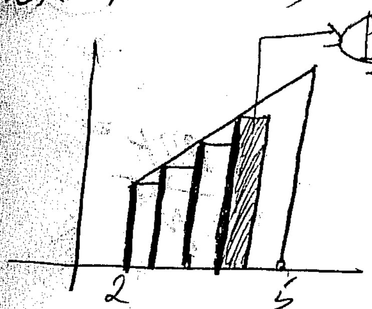
∴ The new definition of area under the curve is ⇒

$$\boxed{\text{AREA} = \lim_{N \to \infty} \sum_{K=1}^{N} f(c_K) \cdot \Delta x}$$

IMP

* when $N \to \infty$ then we cannot draw all N rectangles.

$\Delta x = \frac{5-2}{N} = \frac{3}{N}$



* Since we cannot draw all 'N' rectangles, we pull out an arbitrary rectangle or the K^{th} rectangle.

IMP

∴ $c_1 = 2$

$c_2 = 2 + \Delta x$

$c_3 = 2 + 2\Delta x$

$c_3 = 2 + 3\Delta x$

$c_K = 2 + (K-1)\Delta x$

∴ INSCRIBED RECTANGLE

$$\boxed{c_K = a + (K-1)\Delta x}$$

MOST IMP

CIRCUMSCRIBED RECTANGLE

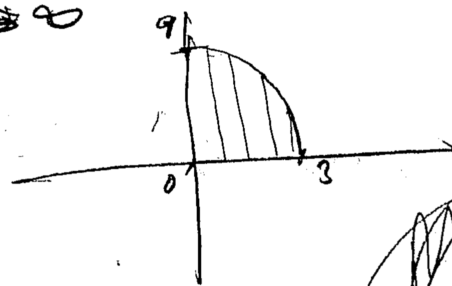
$$\boxed{D_K = a + K\Delta x}$$

If function is increasing then use (left) ~~inscribed~~

if function is decreasing then use (right) ~~inscribed~~

$C = \text{MIN}, \; D = \text{MAX}$

$f(x) = 9 - x^2 \quad [0, 3] \quad N \to \infty$



1). $\Delta x = \dfrac{3}{N}$

2) $C_K = 0 + K \cdot \dfrac{3}{N} = \dfrac{3K}{N}$

3) $f(C_K) = 9 - \left(\dfrac{3K}{N}\right)^2$

$= 9 - \dfrac{9K^2}{N^2}$

SGBALI

MOST IMP

4) Area of the K^{th} $\square$ $= f(C_K) \cdot \Delta x$

$= 9 - \dfrac{9K^2}{N^2}\left(\dfrac{3}{N}\right) = \dfrac{27}{N} - \dfrac{27K^2}{N^3}$

5) $\displaystyle\sum_{K=1}^{N} f(C_K) \cdot \Delta x$

$\displaystyle = \sum_{K=1}^{N} \frac{27}{N} - \frac{27K^2}{N^3} = \sum_{K=1}^{N} \frac{27}{N} - \sum_{K=1}^{N} \frac{27K^2}{N^3}$

N IS A CONSTANT

$\displaystyle = \frac{27}{N} \sum_{K=1}^{N} 1 - \frac{27}{N^3} \sum_{K=1}^{N} K^2$

$= 27 - \dfrac{27}{N^3}\left[\dfrac{N(N+1)(2N+1)}{6}\right]$

$= 27 - \dfrac{(54N^2 + 81N + 27)}{6N^2}$

$= 27 - \dfrac{54}{6} - \dfrac{81}{6N}$

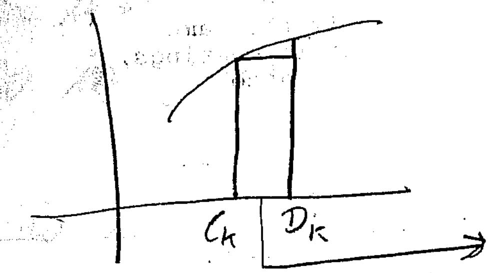
6) $\displaystyle \lim_{N \to \infty} 27 - 9 - \frac{81}{6N} - \frac{27}{N^2} = 18$

THE DEFINITE INTEGRAL

* We know that $A = \lim_{N\to\infty} \sum_{K=1}^{N} f(C_K)\cdot\Delta x$

$$A = \lim_{N\to\infty} \sum_{K=1}^{N} f(D_K)\cdot\Delta x$$

*



COULD USE ANY SPOT x_K^*

$\therefore$ Area $= \lim_{N\to\infty} \sum_{K=1}^{N} f(x_K^*)\cdot\Delta x$

or $\boxed{\underline{\text{AREA}} = \lim_{N\to\infty} \sum_{K=1}^{N} f(x_K^*)\cdot\Delta x_K}$ IMP

& $\boxed{\underline{\text{RIEMANN SUM}} = \sum_{K=1}^{N} f(x_K^*)\cdot\Delta x_K}$ → IMP

No lim

IS THE SAME AS

IMP $\boxed{\text{AREA} = \lim_{\max \Delta x_K \to 0} \sum_{K=1}^{N} f(x_K^*)\cdot\Delta x_K}$ IMP ✓ ✓

* If a function is defined on a closed Interval $[a,b]$ then the DEFINITE INTEGRAL of 'f' from a to b, denoted $\int_a^b f(x)\,dx$, is defined to be

$$\boxed{\int_a^b f(x)\,dx = \lim_{\max \Delta x_K \to 0} \sum_{K=1}^{N} f(c_K^*)\cdot\Delta x_K}$$ MOST IMP

provided the limit exists.

*SHOULD GET NO 'N' on the NUMERATOR. ← IMP

om: hlwpj!jolang (Joan H Lang +1 908 949 0286)
te: Tue Aug 25 10:46 EDT 1992
bject: Department Meetings
: hlwpj!mtdcr!vbl
: sgbali
: !tlr
ntent-Length: 718

ctor,
en we spoke on the phone last Friday, I mentioned to you
at Diane Pero and I would not be able to attend the department
eting on Aug. 27. We are enrolled in a class for people
ining a LAN network which Marc Hornby strongly recommended that we
ke.
en we were in Area 55, it was usually optional for Support Staff
rsonnel to attend technically oriented meetings. Now that we
e a part of your department in QUEST, we were wondering if the same
licy is followed. If you would like us to attend all future meetings,
 course we will do so. However, it is too late for us to cancel
t of this class.
rry if there has been a misunderstanding.

Joan Lang
hlwpj!jolang

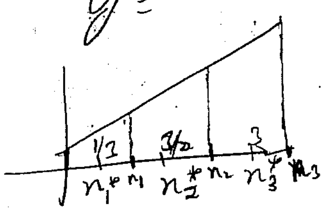
$y = 2x - 5 \quad [2,5] \qquad A = 12$

$\int_2^5 2x - 5\,dx = 12$

$\Delta x = 4/3 \qquad \Delta x = x_N - x_{N-1}$

$y = x+1, \quad a = 0, b = 4, N = 3, \quad x_1^* = 1/3, x_2^* = 3/2, x_3^* = 3$

$x_1 = 1, x_2 = 2, x_3 = 4$



? TO ASK

$$\sum_{K=1}^{N} f(x_K^*)\cdot\Delta x_K = f\left(\tfrac{1}{3}\right)\cdot 1 + f\left(\tfrac{3}{2}\right)\cdot 1 + f(3)\times 2$$

$$= \frac{4}{3} + \frac{5}{2} + 8 = \text{Riemann Sum}$$

MOST IMP

* a) If f is continuous on $[a,b]$, then f is integrable on $[a,b]$

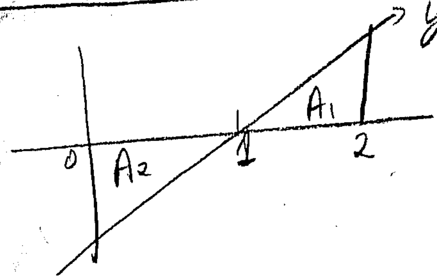
b) If f has only finitely many points of discontinuity on $[a,b]$ and if there is a positive number M such that $-M \le f(x) \le M$ for all x in $[a,b]$ then f is integrable on $[a,b]$.

If f is nonnegative and continuous on $[a,b]$, then

$$\boxed{A = \begin{bmatrix}\text{area under}\\ y=f(x)\\ \text{over } [a,b]\end{bmatrix} = \int_a^b f(x)\,dx}$$

OR

$$\boxed{\int_a^b f(x)\,dx = \begin{bmatrix}\text{area above}\\ [a,b]\end{bmatrix} - \begin{bmatrix}\text{area below}\\ [a,b]\end{bmatrix}}$$

* 

then $\int_0^2 (x-1)\,dx = A_1 - A_2 = 0$

* ~~Do not find the~~ Do not negate the area under the x-axis unless asked for the Area. Just do the Riemann Sum.

THE FIRST FUNDAMENTAL THEOREM OF CALCULUS

States that

* If f is continuous on $[a, b]$ and if F is an anti-derivative of f on $[a, b]$, then

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

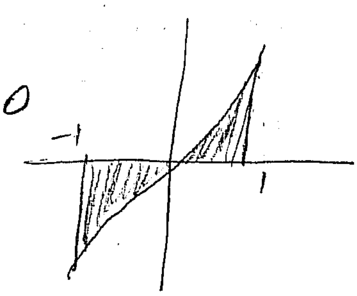
MOST IMP

* $\displaystyle\int_a^b f(x)\,dx = F(x)\Big|_a^b$

* $\displaystyle\int_1^2 x^2\,dx = \frac{x^3}{3}\Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$

$\Rightarrow$ If $\displaystyle\frac{x^3}{3} + c\Big|_1^2 = \frac{8}{3} + c - \frac{1}{3} - c = \frac{7}{3}$

* $\displaystyle\int_{-1}^1 x^3\,dx = \frac{x^4}{4}\Big|_{-1}^1 = \frac{1}{4} - \frac{1}{4} = 0$



* $$\boxed{\int_a^b f(x)\,dx = \left[\int f(x)\,dx\right]_a^b} \longrightarrow \underline{\text{IMP}}$$

* a) $\displaystyle\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$

b) $\displaystyle\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

c) $\displaystyle\int_a^b [f(x)\cdot g(x)]\,dx \neq \int_a^b f(x)\,dx \cdot \int_a^b g(x)\,dx$

* a) If a is in the domain of f, we define

$$\boxed{\int_a^a f(x)\,dx = 0}$$

b) If $b < a$ and if f is integrable on $[b, a]$, then

IMP

$$\boxed{\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx} \rightarrow \text{Very IMP}$$

EVALUATING DEFINITE INTEGRALS BY SUBSTITUTION

* $\int_1^2 x^2\,dx = \int_1^2 t^2\,dt$ because 'x' & 't' are DUMMY VARIABLE.

but * $\int x^2\,dx \neq \int t^2\,dt$ → Indefinite Integral

$$\frac{x^3}{3} + C \neq \frac{t^3}{3} + C$$

* $\int_1^2 (4x-2)^3\,dx$ $\quad u = 4x-2$, $du = 4\,dx$

$$\frac{1}{4}\int_1^2 u^3\,du$$

$$\frac{1}{4}\,\frac{u^4}{4} + C = \frac{u^4}{16} + C \Rightarrow \left.\frac{(4x-2)^4}{16} + C\right|_1^2$$

------------- Begin Forwarded Message -------------
Message-Version: 2
From: !jolang (Joan H Lang +1 908 949 0286)
Date: Tue Oct 13 16:56 EDT 1992
End-of-Header:
Email-Version: 2
Subject: Dept. Mtg. agenda
To: !org=45351:all=y
End-of-Protocol:
Content-Type: text
Content-Length: 676

Members Department 45351:

Reminder: Our next department meeting is on Tuesday, October 20 from 1 p.m. to 4 p.m. in Room 2N-529.

The entire agenda will be dedicated to Norm Shaer who will be our guest speaker. He will discuss such topics as the New Organizational Paradigm, the Golden Thread, etc.
If anyone would like a question answered by Norm at this meeting, we are asking you to please submit them in advance to either Shri Bali (HO Room 2K-402, 949-0281, hostar!sgbali) or me (Joan Lang, HO Room 2K-426, 949-0286, hostar!jolang) by this FRIDAY, OCTOBER 16.
We will then submit all of the questions to Norm in advance of the meeting.
Thank you.
Joan

* $\int_1^2 (4x-2)^3\,dx$ $\quad u = 4x-2$, $du = 4\,dx$

$\frac{1}{4}\int_2^6 u^3\,du$

$a = 4-2 = 2$, $b = 8-2 = 6$ } IMP

$= \frac{1}{4}\frac{u^4}{4}\Big|_2^6 = \frac{u^4}{16}\Big|_2^6$

* DO NOT CONVERT u back into x, only in Indefinite integrals.

MOST IMP

* $\int_{-\frac{3\pi}{4}}^{-\pi/4} \sin x \cos x\,dx$ $\quad u = \sin x$, $du = \cos x\,dx$

$\sin -\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$\sin -\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\Rightarrow \int_{-\frac{\sqrt{2}}{2}}^{-\frac{\sqrt{2}}{2}} u\,du$

$= \frac{u^2}{2}\Big|_{-\frac{\sqrt{2}}{2}}^{-\frac{\sqrt{2}}{2}}$

$\Delta x = \frac{b-a}{x}$ not $\frac{b-a}{n}$

* $\int_0^{\pi/4} \frac{\cos 2x}{\sqrt{7-3\sin 2x}}$ $\quad u = 7-3\sin 2x$

$du = -6\cos 2x \, dx$

$-\frac{1}{6}\int_{7}^{4} \frac{du}{\sqrt{u}} = -\frac{1}{6}\int_7^4 u^{-1/2}\, du$

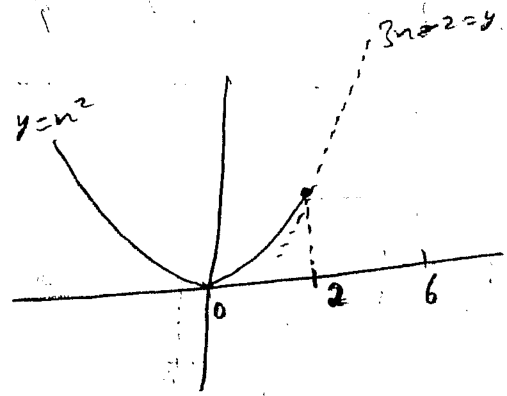
$\Rightarrow -\frac{1}{6}\, 2u^{1/2}\Big|_7^4 = -\frac{1}{3} u^{1/2}\Big|_7^4$

IMP PROBLEM

$$\boxed{\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx} \rightarrow \text{IMP}$$

$$f(x) = \begin{cases} x^2, & x \leq 2 \\ 3x-2, & x \geq 2 \end{cases}$$

$\int_0^6 f(x)\,dx$



* f is contd. as $\lim_{x\to 2^-} x^2 = 4 = \lim_{x\to 2^+} 3x-2 = 4$

* $\therefore \int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx$

* $\int_{-2}^4 |x+1|\,dx \Rightarrow \begin{cases} x+1, & x \geq -1 \\ -(x+1), & x < -1 \end{cases}$

$= \int_{-2}^{-1} (-x-1)\,dx + \int_{-1}^{4} (x+1)\,dx$

MEAN-VALUE THEOREM

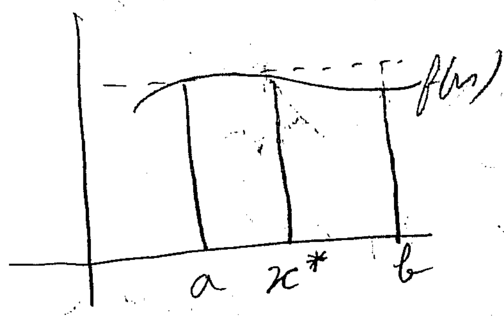
For Differentiation — contd. & diff & $f'(c) = \frac{f(b)-f(a)}{b-a}$

For Integration ⇒ AVERAGE VALUE or the MEAN VALUE.

* If f is continuous on [a,b], then there exists at least one number x^* in [a,b]

$$\ni \boxed{\int_a^b f(x)\,dx = f(x^*)\cdot(b-a)} \to \text{IMP}$$

* $$\boxed{f(ave) = \left(\frac{1}{b-a}\right)\left(\int_a^b f(x)\,dx\right)} \to \text{IMP}$$



* Area under curve =

$$\int_a^b f(x)\,dx = f(x^*)\cdot(b-a)$$

IMP Bob

* $f(x) = x^2$; [1,4]

a) find f(ave)

b) find x^* or $x \in [1,4]$ that's guaranteed by the mean value theorem for Integrals

a) $f(ave) = \frac{1}{(b-a)}\left(\int_a^b f(x)\,dx\right) = \frac{1}{3}\frac{x^3}{3} \cdot \frac{1}{3}\frac{x^3}{3}\Big|_1^4 = \frac{1}{9}[4^3-1] = 7$

b) $f(x^*) = 7$, $(x^*)^2 = 7 \Rightarrow x^* = \pm\sqrt{7} \Rightarrow x^* = \sqrt{7}$ in [1,4]

THE SECOND FUNDAMENTAL THEOREM OF CALCULUS

MOST IMP

AVERAGE VALUE —

$$\boxed{f(x^*) = f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

$$\boxed{V_{ave}} = \frac{s(t_1) - s(t_0)}{t_1 - t_0} = \frac{1}{t_1 - t_0}\int_{t_0}^{t_1} v(t)\,dt$$

$$\boxed{a_{ave} = \frac{1}{t_1 - t_0}\int_{t_0}^{t_1} a(t)\,dt} = \frac{v(t_1) - v(t_0)}{t_1 - t_0}$$

IMP

* Let f be continuous on an open interval I and let a be any point in I. If F is defined by

$$\boxed{F(x) = \int_a^x f(t)\,dt} \rightarrow \text{IMP}$$

then $\boxed{F'(x) = f(x)}$ at each point x in the interval I. → IMP

→ REMEMBER TO REPLACE FOR x

$$F(x) = \int_2^x \sqrt{3t^2+1}\,dt \qquad F(2) = \int_2^2 \sqrt{3t^2+1}\,dt = 0$$

$$F'(x) = \sqrt{3x^2+1} \qquad F'(2) = \sqrt{13}$$

$$f(x) = \sqrt{3t^2+1}$$

$$F''(x) = \frac{1}{2}(3x^2+1)^{-1/2}\cdot 6x$$

$$F''(2) = \frac{1}{2}(13)^{-1/2}\cdot 12 = \frac{6}{\sqrt{13}}$$

DISPLACEMENT

TOTAL DISTANCE

IMP

$$\boxed{\int_{t_1}^{t_2} |v(t)|\,dt} \Rightarrow \text{TOTAL DISTANCE TRAVELLED BY THE PARTICLE}$$

$$\int_{t_1}^{t_2} v(t)\,dt \Rightarrow \text{DISPLACEMENT}$$

MOST IMP

$F(x) = \int_{\pi/4}^{x} \cos 2t\,dt$

$F'(x) = \cos 2x$

INTEGRATE FIRST & THEN DIFFERENTIATE TO PROVE 2nd F.T.C.

$\int_{\pi/4}^{x} \cos 2t = \sin 2t + c$

$\frac{d}{dx} \sin 2x = \cos 2x$

$$\boxed{\frac{d}{dx}\int_{a}^{g(x)} f(t)\,dt = f(g(x))\,g'(x)}$$

V.V IMP

$v(t) = 2t^3 - 2t$

$2t(t^2 - 1)$

+ + + , − − − , + + +
−1 1

− − − , + + + +
0

− − , + + + , − − − , + + +
−1 0 1

$t = 0 - 5$

$x(0) =$

$x(1) =$

$x(5) =$

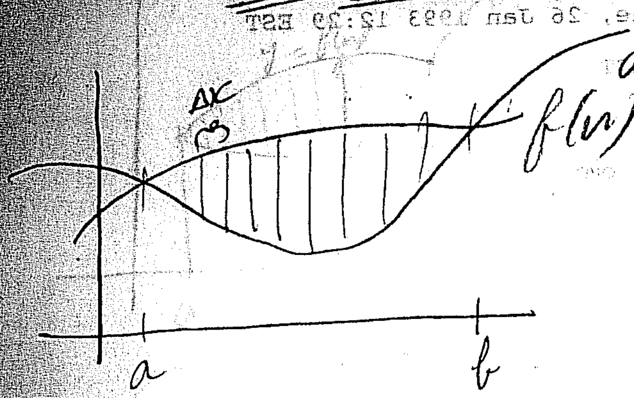
...sage-Version: 2
...e: Fri Oct 16 09:51:31 1992
...-of-Header:
...ll-Version: 2
...ne: 908-949-0281
Message-ID: <winPMXSTAR-2.2.1b-sgball-XXXXXXXXX-239>
...ject: Re: PC/LAN Course
...march
Reply-To: Your message <PMX-LAN-2.2.1-*****-hostar-march-1882> of Fri Oct 16
...-of-Protocol:
...tent-Type: Text
...tent-Length: 1146

...uld we take this a group, or even jointly with Tom's group?

6. APPLICATIONS OF THE DEFINITE INTEGRALS

6.1 AREA BETWEEN TWO CURVES

FIRST AREA PROBLEM

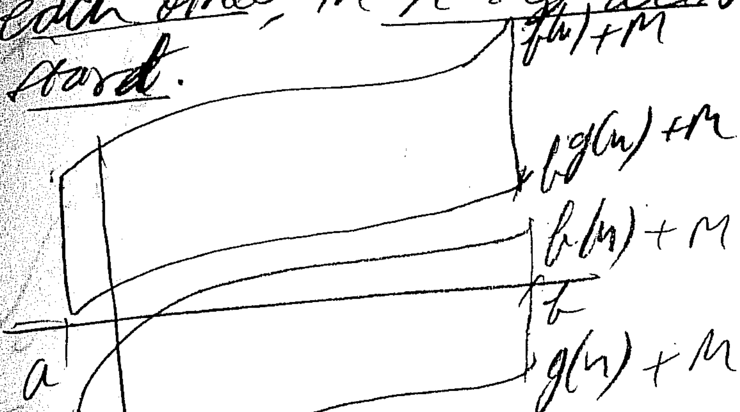


$N \to \infty$

* Height of the $\square$'s $= f(x) - g(x)$
* Area of the rectangles $= \lim_{N\to\infty} \sum_{k=1}^{N} [f(x) - g(x)] \cdot \Delta x$

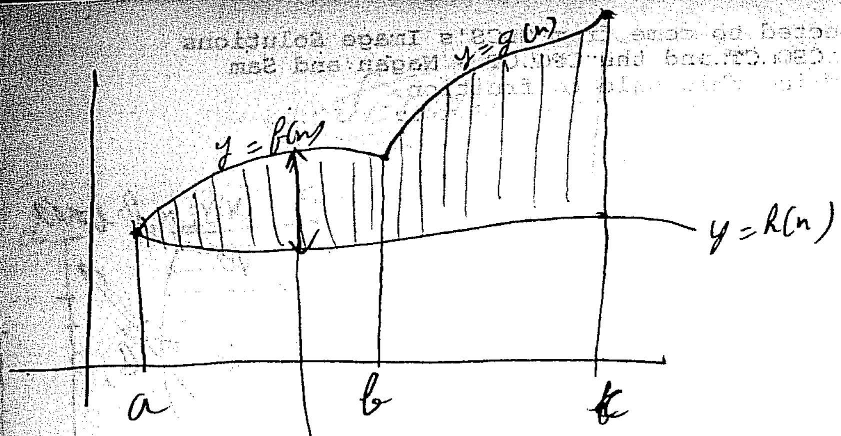
or $\int_a^b f(x) - g(x)\, dx$

* Always find out where the curves intersect each other, the x & y axis & where they end & start.



$$\text{Area} = \int_a^b (f(x) + M) - (g(x) + M)\, dx$$

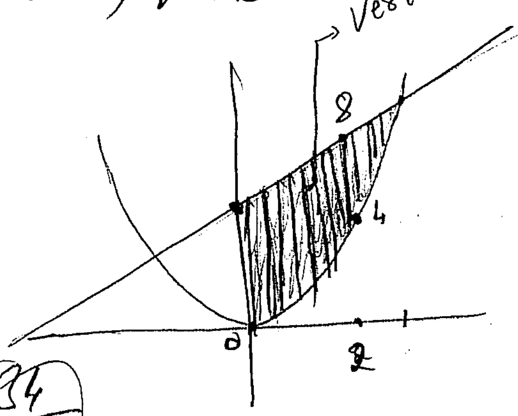
$$= \int_a^b f(x) - g(x)\, dx$$



THE ARBITRARY ~~VERTICAL BAR~~ changes function as it moves along the graph indicating if the function has to be done in 2 ~~parts~~ or more or one part.

$$\int_a^b f(x) - h(x)\,dx + \int_b^c g(x) - h(x)\,dx$$

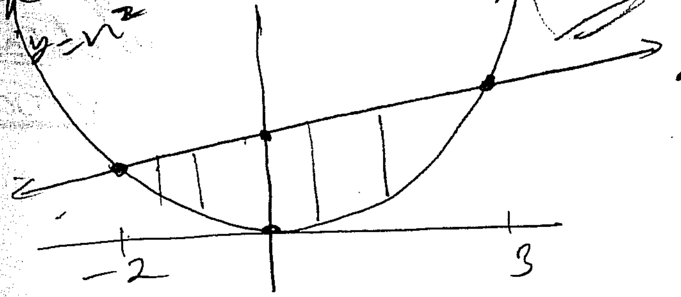
* $f(x) = x+6 \rightarrow$ above $x=0,\ x=2$
$g(x) = x^2 \rightarrow$ below



$$\int_0^2 x+6 - x^2\,dx$$

$$= \frac{x^2}{2} + 6x - \frac{x^3}{3}\Big|_0^2 = \frac{34}{3}$$

*



$$\int_{-2}^3 x+6 - x^2\,dx$$

$$= \frac{x^2}{2} + 6x - \frac{x^3}{3}\Big|_{-2}^3 =$$

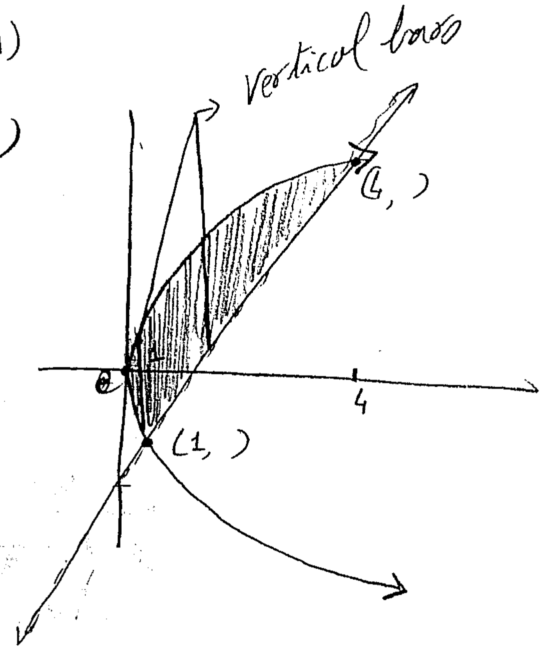
* Only the limits of the area have been altered which changes the area.

nding for this project is expected to come from GBCS's Image Solutions
ganization (Prestinario). The CSG CT and the CPG CT - Nagan and Sam
opello will work together to bring this sale to fruition.

gan Raman
EST GAM for CSG/BCS

* $x = y^2 \rightarrow$ FUNCTION OF Y (1)

$y = x - 2$ ——— (2)



$= x = y^2,\ y = x - 2$

$\therefore x = (x-2)^2$

$x = x^2 - 4x + 4$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$\therefore x = 4, 1$ → Points of intersection of the 2 curves.

* Convert (1) given as a function of y to an equation given as a function of x.

$\therefore$ $y = \pm\sqrt{x}$ ——— (1)

$y = x - 2$ ——— (2)

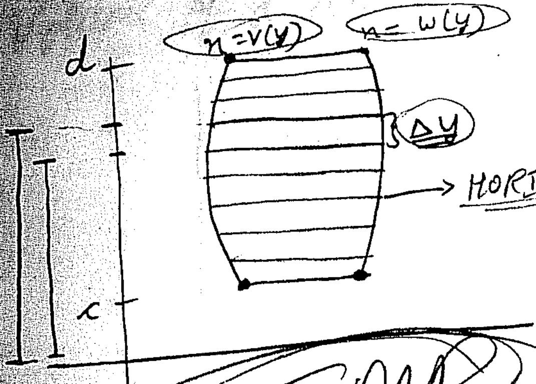
$$\therefore \int_0^1 \sqrt{x} - (-\sqrt{x})\, dx + \int_1^4 \sqrt{x} - (x-2)\, dx$$

$\sqrt{x}$ ↓ TOP OF PARABOLA

$(-\sqrt{x})$ ↓ BOTTOM OF PARABOLA

↳ shows that eq. ~~has~~ is given as a function of x & so doe using VERTICAL LINE

SECOND AREA PROBLEMS



→ HORIZONTAL RECTANGLES

$$\int_c^d w(y) - v(y)$$

$w(y)$: RIGHT SIDE — $v(y)$: LEFT SIDE

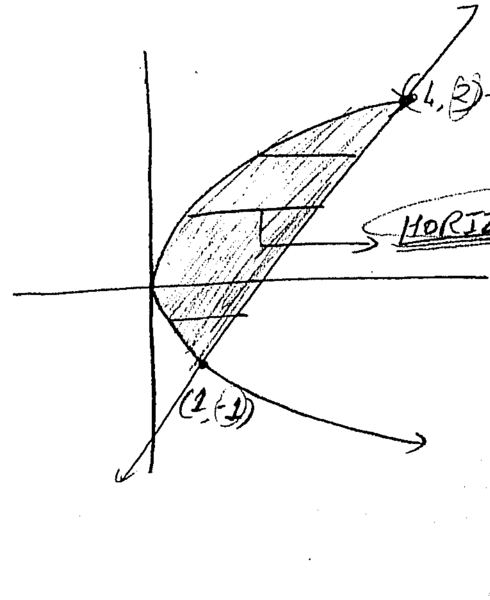
MOST IMP

TO ADD UP VERTICAL BARS THE FUNCTION MUST BE A FUNCTION OF x AND TO ADD UP HORIZONTAL BARS, THE FUNCTION MUST BE A FUNCTION OF y

$x = y^2$

$y = x-2 \Rightarrow x = y+2$

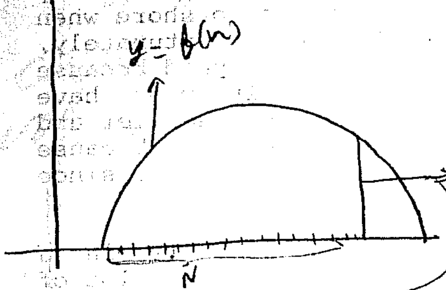
$$\int_{-1}^{2} y+2-y^2 \, dy$$



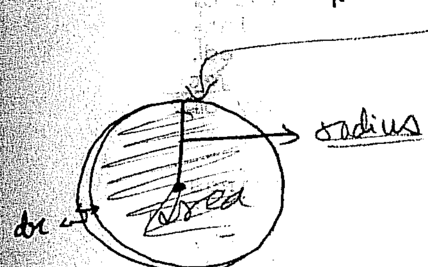
Volume

VOLUMES

* Volume of a Box = $l \times w \times h$



radius of one slice
(take out one slice)
(one circle)



Area of $\bigcirc = \pi r^2$

& $\boxed{r = f(x)}$

$\therefore$ Area $= \pi f(x)^2$

Volume of one slice $= \pi f(x)^2 \cdot dx$

or $V_k = \pi f(x_k)^2 \cdot \Delta x_k$

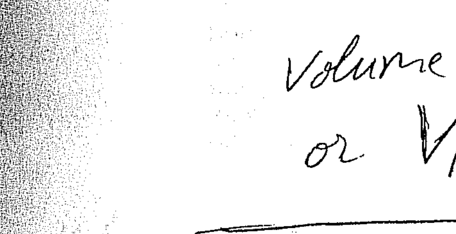
↳ any slice

$$\boxed{\lim_{\max \Delta x_k \to 0} \pi \sum_{k=1}^{N} f(x_k^*)^2 \cdot \Delta x_k}$$

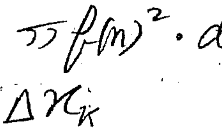
* Δx keeps on changing as $N \to \infty$ & $\max \Delta x_k \to 0$

or $\boxed{V = \pi \int_a^b f(x)^2 dx}$ → METHOD OF DISK

* Use method of disk if the graph lying on the x-axis and revolving around it.



radius of the circular slice. → dy



* Here width = dy & not dx

$\boxed{r = w(y)}$

Area $= w(y)^2 \cdot \pi$

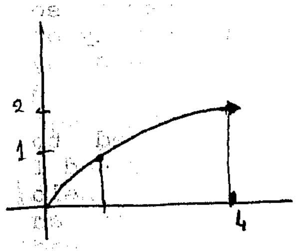
$$\boxed{\lim_{N \to \infty} \pi \sum_{k=1}^{N} w(y_k)^2 \cdot \Delta y_k}$$

$$\boxed{\text{Volume} = V_k = \pi \int_c^d w(y)^2 dy}$$

METHOD OF DISK → { REVOLUTION AROUND X-AXIS → dx } IMP → $y=0$

V.V. IMP { REVOLUTION AROUND Y-AXIS → dy } IMP → $x=0$

* Volume of a solid obtained when the region under the curve $y=\sqrt{x}$ over the interval $[1,4]$ is revolved around the x-axis



→ DISK PROBLEM

$$\pi\int_1^4 f(x)^2\,dx$$

$$= \pi\int_1^4 (\sqrt{x})^2\,dx$$

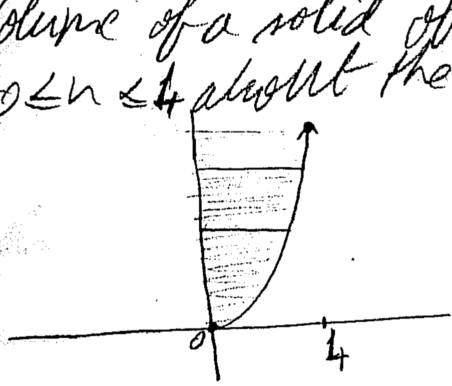
$$= \pi\,\frac{x^2}{2}\Big|_1^4 = \pi\left(8-\frac{1}{2}\right)$$

$$= \frac{15\pi}{2}$$

* Volume of a solid obtained by rotating $y=x^2$ for $0 \le x \le 4$ about the y-axis : IMP ← $\boxed{y=(4)^2=16}$

$x = \pm\sqrt{y}$

$\therefore x = +\sqrt{y}$



$$\pi\int_0^1 (\sqrt{y})^2\,dy$$

$$= \pi\,\frac{y^2}{2}\Big|_0^{16} = \pi\left(\frac{256}{2}\right)$$

$$= 128\pi$$

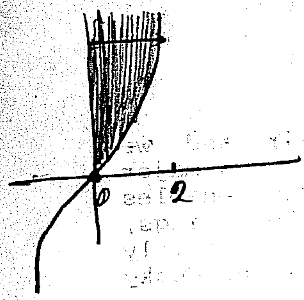
V.V. IMP { * DO NOT FORGET TO CHANGE THE LIMITS IF THE PROBLEM IS GIVEN IN TERMS OF Y OR ~~AS~~ X AS A FUNCTION OF Y. }

V.V. IMP * DO NOT FORGET TO PUT THE ORIGINAL EQUATION IN TERMS OF Y.

IMP

* $y = x^3$ on $[0,2]$, y-axis $\quad x = \pm\sqrt[3]{y}$

or $x = +\sqrt[3]{y}$ $\quad [0,2]$ = positive only

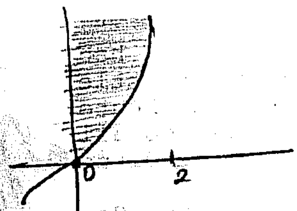


$$\pi\int_0^8 (\sqrt[3]{y})^2\, dx$$

$$= \pi\int_0^8 y^{2/3}\, dx = \pi\, \frac{3}{5}\, y^{5/3}\Big|_0^8$$

$$= \pi\left(\frac{96}{5}\right) = \frac{96\pi}{5}$$

* $y = x^3$ on $[0,2]$, x-axis

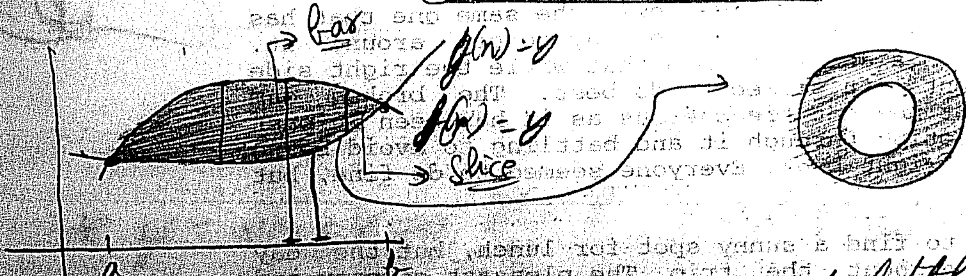


$$\pi\int_0^2 (x^3)^2\, dx$$

$$= \pi\int_0^2 x^6\, dx = \pi\, \frac{x^7}{7}\Big|_0^2$$

$$= \pi\, \frac{128}{7} = \frac{128\pi}{7}$$

METHOD OF WASHERS



top − bottom → IMP

\# Area of big circle − Area of little circle

(i) Radius of big circle $= \pi r^2 = \pi f(x)^2$

(ii) Radius of little circle $= \pi r^2 = \pi g(x)^2$

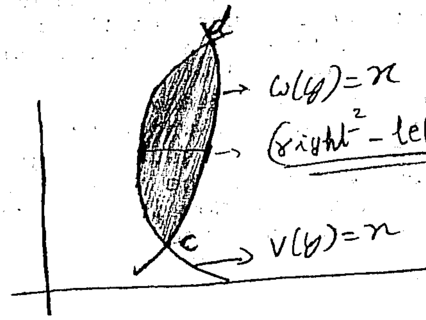
$(\text{top of the bar})^2 - (\text{bottom of the bar})^2$

$$\boxed{\text{Volume} = \lim_{\max(\Delta x_K)\to 0} \pi \sum_{K=1}^{N} f(x_K)^2\,\Delta x_K - g(x_K)^2\,\Delta x}$$

or

$$\boxed{V_K = \pi \int_a^b f(x)^2 - g(x)^2\,dx}$$

or



* Remember to find the intersection points of 2 graphs for the limits IMP

* (Right of the bar)^2 − (left of the bar)^2

$$\boxed{V = \lim_{\max \Delta x_K \to 0} \pi \sum_{K=1}^{N} \omega(y)^2 \cdot \Delta y - v(y)^2 \cdot \Delta y}$$

or

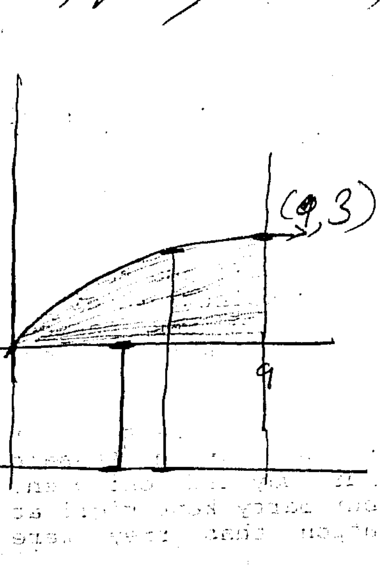
$$\boxed{V_K = \pi \int_c^d \omega(y)^2 - v(y)^2\,dy}$$

V.V.V. IMP

Rotation around a given line

* $y = \sqrt{x}$, $y = 0$, $x = 9$, Find V when rotating around $y = -2$

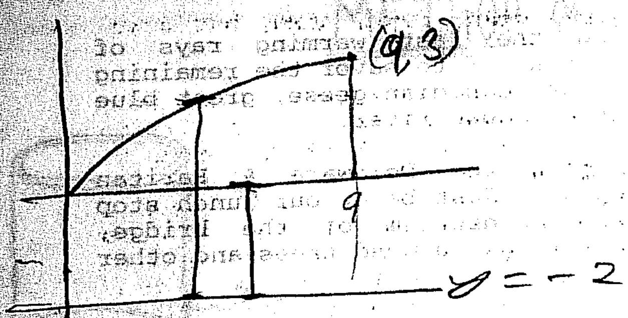
MOST IMP



MOST IMP PART

Now the center $\Rightarrow y = -2$

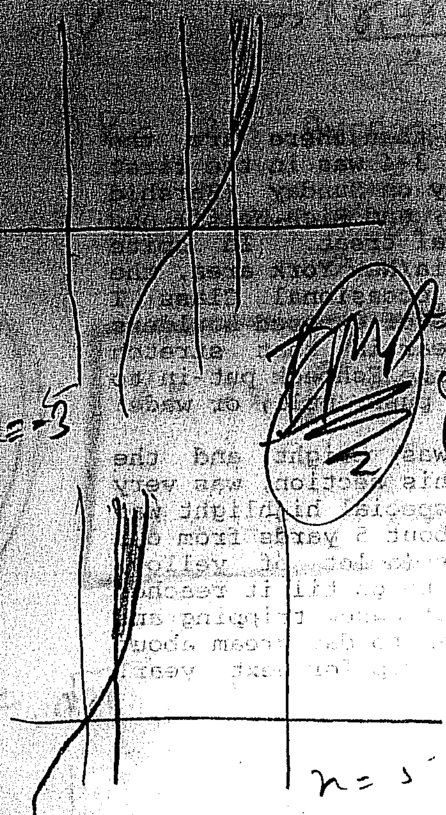
$$\pi \int \underbrace{[\sqrt{x} - (-2)]^2}_{\text{radius of big circle}} - \underbrace{[(-2)]^2}_{\text{radius of little circle}}$$



∴ Now the center ⇒ y = -2

$$\pi\int [\sqrt{x} - (-2)]^2 - [(-2)]^2$$

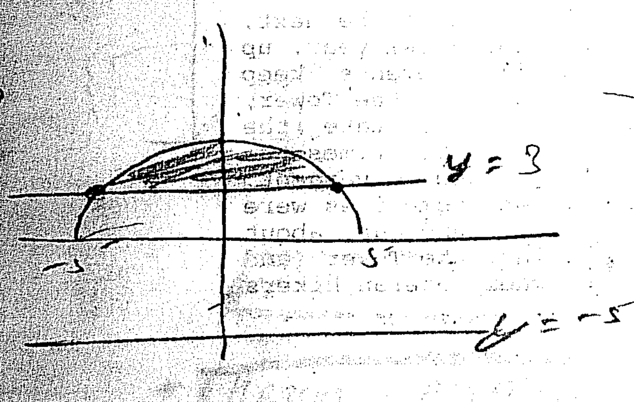
IF THE AXIS OF ROTATION IS PARALLEL TO Y-AXIS, THEN IT IS A dy PROBLEM. IF THE AXIS OF ROTATION IS PARALLEL TO X-AXIS, THEN IT IS A dx PROBLEM.



$$\pi\int [(y-1)^{1/3} - (-3)]^2 - [1-(-3)]^2\,dy$$

RECOMMEND WASHERS & NO SHELLS

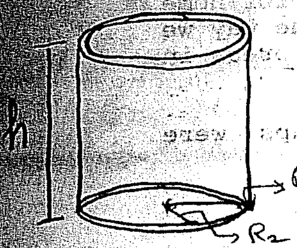
$$\pi\int [(5-1)]^2 - [5-(y-1)^{1/3}]^2\,dy$$



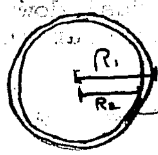
$$\pi\int_{-4}^{4} (\sqrt{25-x^2})^2 - (3)^2\,dx$$

$$\pi\int_{-4}^{4} [\sqrt{25-x^2} - (-5)]^2 - [3-(-5)]^2$$

METHOD OF SHELLS



Bottom of the shell



dy or dx (Thickness)

$\therefore$ AREA $= \pi r_1^2 - \pi r_2^2$

volume = area $\cdot h$

$V = (\pi r_1^2 - \pi r_2^2) \cdot h$

$V = \pi (r_1^2 - r_2^2) \cdot h$

$V = \pi [(r_1 - r_2)(r_1 + r_2)] \cdot h$

$V = 2\pi \left(\frac{r_1 + r_2}{2}\right) \cdot (r_1 - r_2) h$

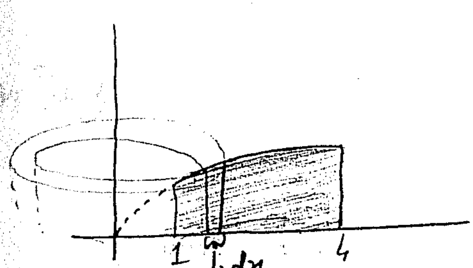
V = 2π . AVERAGE RADIUS . THICKNESS . height

$V = 2\pi \int_a^b$ ave. radius . height or dx (thickness)

MOST IMP

$$\boxed{\begin{array}{l} V = 2\pi \int_a^b \text{AVERAGE RADIUS} \cdot \text{HEIGHT} \cdot dx\ [\text{THICKNESS}] \\ \qquad \text{OR} \\ V = 2\pi \int_c^d \text{AVERAGE RADIUS} \cdot \text{HEIGHT} \cdot dy\ [\text{THICKNESS}] \end{array}}$$

* $y = \sqrt{x}$, $x = 1$, $x = 4$, Around y-axis



$\therefore V = 2\pi \int_1^4 \underbrace{x}_{\text{ave. rad.}} \cdot \overbrace{\sqrt{x}}^{\text{height}} \; \overbrace{dx}^{\text{thickness}}$

$\boxed{x \text{ IS THE ARBITRARY RADIUS}}$

* $\boxed{\text{HEIGHT} = \text{RIGHT} - \text{LEFT OR TOP} - \text{BOTTOM}}$

(i) IF THE SHELLS ARE PLACED AROUND THE <u>Y-AXIS</u>, THEN, THROUGH THE METHOD OF SHELLS IT IS A <u>dx</u> PROBLEM.

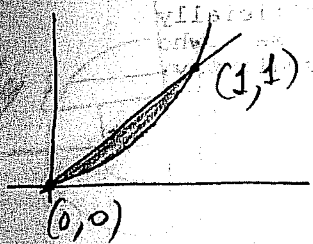
(ii) IF THE SHELLS ARE PLACED AROUND THE <u>X-AXIS</u>, THEN, THROUGH THE METHOD OF SHELLS IT IS A <u>dy</u> PROBLEM.

* $y = x^2$, $y = x$ $\Rightarrow$ ① $x = \pm\sqrt{y}$ & $x = y$ ②

$x^2 - x = 0$

$x(x-1) = 0$

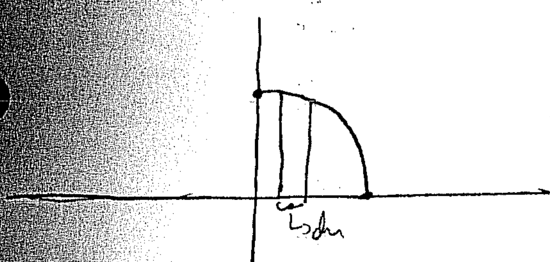
$x = 0, 1$



$V = 2\pi \int_0^1 y\,[\underbrace{\sqrt{y} - y}]\,dy$

↳ Height (right - left)

* $y = -x^2 + 1$, around y-axis

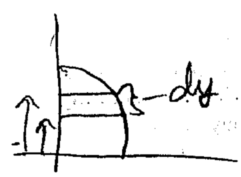


$V = 2\pi \int_0^1 x(-x^2+1)\,dx$

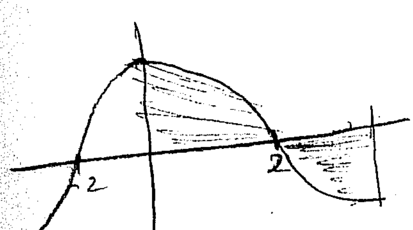
* around x-axis

$x = \pm\sqrt{1-y}$

$2\pi \int_0^1 y(\sqrt{1-y})\,dy$



* $x^2 + y^3 = 4$, $x = 0$, $x = 4$, $y = 0$

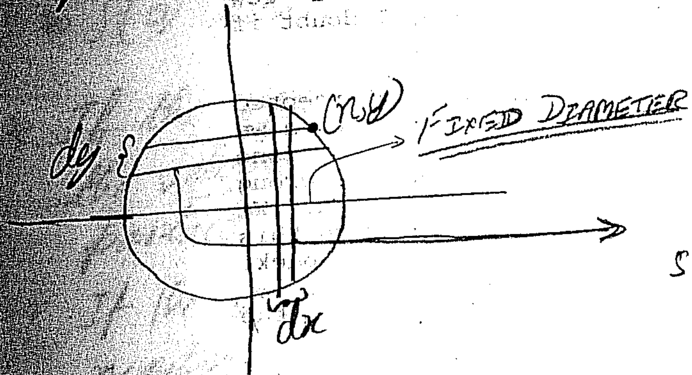


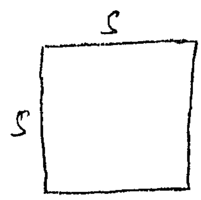
* IF THE

METHOD OF SLICES

VOLUME OF SLICES

* Find the ~~Volume~~ Area of a solid whose base is a circle of radius = 2 m, If all the cross section to a fixed diameter of the base are square.





IN TERMS OF y

Area = $\int s^2 dy$

↳ HAS TO BE IN TERMS OF y

$s = 2x$

$s^2 = 4x^2$

$\therefore \int 4x^2 dy$

EQUATION OF A CIRCLE = $x^2 + y^2 = r^2$

$x^2 = 4 - y^2$

$x = \pm\sqrt{4 - y^2}$

$$\int_{-2}^{2} 4(4 - y^2)\,dy = \frac{128}{3}$$

* TRY TO GET THE EQUATION IN TERMS OF y or dx. → IMP

MOST IMP

~~MOST IMP SECTION~~ RECTILINEAR MOTION

$v(t) = s'(t) = \frac{ds}{dt}$ & $a(t) = v'(t) = \frac{dv}{dt}$

$\boxed{s(t) = \int v(t)\,dt}$ & $\boxed{v(t) = \int a(t)\,dt}$

* $s(0)$ can be written as s_0 or $v(0)$ as v_0
* If the velocity is positive and so is the acceleration or if the velocity is negative & so is the acceleration then the particle is speeding up * PARTICLES AT REST = C.P
* If the velocity is ~~specti~~ positive and the acceleration is negative or vice-versa, then the particle is slowing down.
* Sign Line of a factored function shows where the function is above or below the x-axis.
* MOTION NEAR THE EARTH'S SURFACE — An object moving on a vertical line near the earth's surface and subject only to the force of gravity moves with constant acceleration. This constant (g) is $\boxed{32\ ft/sec^2}$ or $\boxed{9.8\ m/sec^2}$
* ACCELERATION OF A PARTICLE IS ALWAYS NEGATIVE

$$\boxed{a(t) = -g}$$

$$v(t) = \int a(t)\,dt = \int -g\,dt = -gt + C_1$$

when $t=0$ then $v_0 = -g(0) + C_1$

$$v_0 = C_1$$

$$\boxed{v(t) = -gt + v_0}$$ v_0 = Constant

EXTREMELY IMP

$$S(t) = \int v(t)\,dt = \int -gt + v_0\,dt$$

$$= -\frac{1}{2}gt^2 + v_0 t + C_2$$

when $t = 0$ then $S_0 = -\frac{1}{2}(0)t^2 + v_0(0) + C_2 = C_2$

$S_0 = C_2$

$$\therefore \boxed{S(t) = -\frac{1}{2}gt^2 + v_0 t + S_0} \rightarrow \text{EXTREMELY IMP}$$

$$\boxed{\text{SPEED} = |\text{VELOCITY}|} \rightarrow \text{IMP}$$

The DISPLACEMENT of a particle can be found by → IMP

$$\boxed{\int_{t_1}^{t_2} v(t)\,dt = S(t)\Big|_{t_1}^{t_2}} = \text{DISPLACEMENT on } [t_1, t_2]$$

- If the $v(t) > 0$, then the particle moves in the +ive direction only & so the displacement is the total distance travelled.
- If the $v(t) < 0$, then the particle moves in the -ive direction only so the displacement is the total distance travelled.
- when the travelling particle changes direction then we

$$\boxed{\int_{t_1}^{t_2} |v(t)|\,dt = \text{TOTAL DISTANCE TRAVELLED on } [t_1, t_2]} \rightarrow \text{IMP}$$

OR

BETTER METHOD

(i) SIGN LINE THE VELOCITY $v(t) = (t-4)(t-2)$ $0 \le t \le 5$

(ii) FIND THE VALUE OF THE POSITION FUNCTION AT THE VARIOUS CRITICAL PTS. $S(0)$, $S(2)$, $S(4)$, $S(5)$ & not $v(0)$, $v(2)$ etc

(iii) ADD THE $S(t)$ VALUES

IMP

7. LOGARITHM AND EXPONENTIAL FUNCTIONS

INVERSE FUNCTIONS

Exponents & logs are inverse functions.

In Inverse functions DOMAIN & RANGE ARE SWITCHED → IMP

$f = \{(1,2)\ (3,4)\ (5,6)\}$

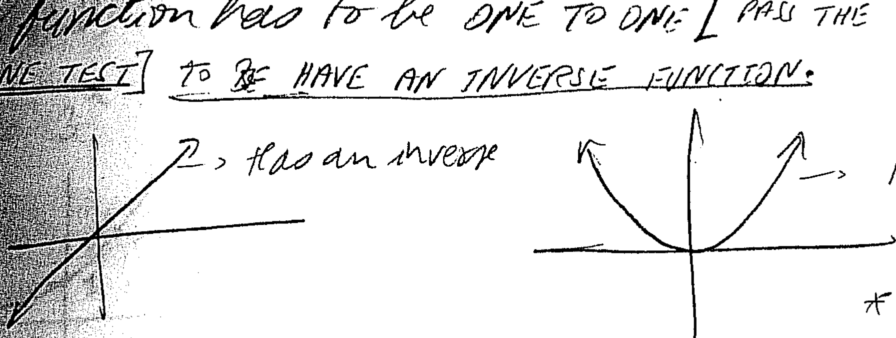
$g = \{(2,1)\ (4,3)\ (6,5)\}$

$f = \{(1,2)\ (2,2)\ (3,2)\}$

$g = \{(2,1)\ (2,2)\ (2,3)\}$ → not a function.

* Every function does not have an Inverse function.

A function has to be ONE TO ONE [PASS THE STRAIGHT LINE & THE HORIZONTAL LINE TEST] TO ~~BE~~ HAVE AN INVERSE FUNCTION.



* could make it invertible by LIMITING THE DOMAIN

TO FORM AN INVERSE ALGEBRAICALLY, WE SWITCH THE x's AND THE y's & SOLVE FOR y → V.V. IMP

* f^{-1} → SYMBOL FOR INVERSE.

$y = \frac{x-2}{x} = f(x)$

$x = \frac{y-2}{y}$

$xy = y - 2$

$xy - y = -2$

$y(x-1) = -2$

$y = \frac{-2}{x-1} = f^{-1}(x)$

* $y = x^2 = f(x)$ $\quad D = x \geq 0$, $R = y \geq 0$

$x = y^2$

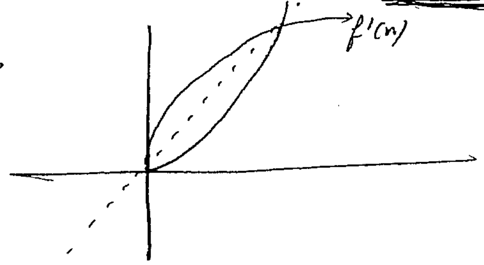
$y = \pm\sqrt{x} = f^{-1}(x)$

$= f^{-1}(x) = \sqrt{x}$ & NO NEGATIVE AS THE DOMAIN OF $f(x)$ IS THE RANGE OF $f^{-1}(x)$

our 32 bodies along the trail in a long

X 9899-29013 edsel!srm
X 9899-93360 hoqax!lent
X 9899-97685 hogpa!peru
X 9902-16729 hogpa!arg
X 9902-80654 aloft!aws
X 9902-84772 edsel!rrb
X 9907-11851 neptune!wjl
X 9907-11853 hogpa!jiml
X 9907-26952 hogpa!scw
X 9907-28539 iexist!opdyke
X 9907-83726 clockwise!let
X 9908-37795 hogpa!jcj1
X 9908-96444 hrmso!nvk
X 9909-20845 research!jcl
X 9909-21340 lcuxlp!jeb
X 9909-28816 zeppo!jhb
X 9912-11361 hrcms!jjc
X 9912-21670 violin!rayc
X 9912-26249 qsun!bfm
X 9912-84898 hogpa!ncw
X 9912-97157 hotlg!mcg
X 9915-24978 r51ux!fsf
X 9917-16125 hoqax!kate
X 9917-36426 longs!mab
X 9917-85666 aluxpo!pl
X 9917-92044 homxc!1358amy
X 9918-12027 mink!aymang
X 9918-19932 physics!ayb
X 9919-18828 gummo!gmy
X 9919-21736 aluxpo!krd
X 9919-81530 homxb!ajp
X 9919-83841 aluxpo!dpl
X 9922-21343 aluxpo!k3t
X 9924-83382 aloft!jean
X 9927-19241 probe!rfb
X 9927-21052 ulysses!gsf
X 9927-27846 aluxpo!chh
X 9927-27848 ihlpy!jth
X 9927-28617 boron!vmp
X 9927-29766 hocpb!dcj
X 9927-30201 hogpa!kurt
X 9927-31086 edsel!koka
X 9927-39974 buckaroo!richard
X 9927-94325 taz!cfgjr
X 9928-34147 wmsa!ram
X 9928-36803 physics!apollo
X 9929-11162 mtnet1!cbm
X 9929-11503 soporo!jjm
X 9929-11637 hosv1!tms2
X 9929-17414 ulysses!rjy
X 9929-24235 hogpa!jj1028
X 9929-27337 germany!davew
X 9929-28358 aluxpo!terry
X 9929-30949 globe1!scm7
X 9929-95550 mtung!mdb
X 9932-31871 mink!me
X 9932-84049 buckaroo!dany
X 9937-34736 cbzoo!cla
X 9937-35861 lzsc!rlb
X 9937-86499 cblph!lmg
X 9938-89969 houxa!yury
X 9938-95422 floyd!kurt
X 9939-16322 whamg!sxb
X 9939-22040 hoqaa!dhg
X 9939-32906 aluxpo!tld
X 9939-34856 aloft!crm
X 9939-82443 whamt!bpd
X 9939-94669 ihwpg!nxd
X 9942-18404 hoqub!ait
X 9944-99318 woomera!pal
X 9947-12050 mozart!slv
X 9947-13712 alux1!cws
X 9947-20692 hogpa!kitko

f(n) → IDENTITY LINE → IMP



* $\boxed{f \circ f^{-1}(n) = f(f^{-1}(n)) = x}$ → IMP

IF FUNCTIONS ARE STRICTLY INCREASING OR DECREASING, AN
THE FIRST DERIVATIVE IS EITHER COMPLETELY POSITIVE OR
COMPLETELY NEGATIVE FOR ALL VALUES OF ~~x~~, THEN THE FUNCTI
IS ONE TO ONE OR IS INVERSIBLE.

eg - * $f(n) = n^3 - 3n^2 + 3n - 1$

$f'(n) = 3n^2 - 6n + 3$

$3(n^2 - 2n + 1)$

$3(n-1)(n-1)$

$3(n-1)^2$

+ + + + + + + +
1
↳ Strictly Increasi
Is Inversible

→ There are 2 ways to find the inverse of $f(n) = n^3 - 3n$

a) $n = y^3 - 3y^2 + 3y - 1$

$\Rightarrow 1 = 3y^2 \frac{dy}{dn} - 3 \cdot 2y \frac{dy}{dn} + 3 \frac{dy}{dn}$

$\Rightarrow 1 = \frac{dy}{dn}(3y^2 - 6y + 3)$

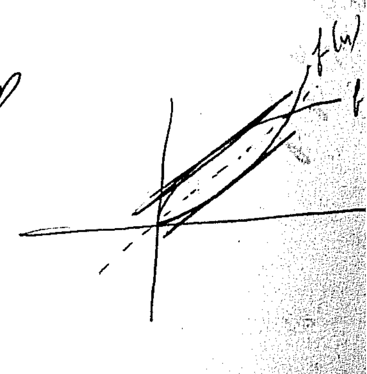
$\Rightarrow \frac{dy}{dn} = \frac{1}{3y^2 - 6y + 3}$

b) $\boxed{\frac{dy}{dn} \text{ of INVERSE} = \frac{1}{dn/dy}}$ → IMP

$n = y^3 - 3y^2 + 3y - 1$

$\frac{dn}{dy} = 3y^2 - 6y + 3$

$\frac{dy}{dn} = \frac{1}{3y^2 - 6y + 3}$



$f(n) = 2n+3$ $\qquad f^{-1}(n) = \frac{n-3}{2}$

$f'(n) = \underline{\underline{2}}$ $\qquad = \frac{1}{2}n - \frac{3}{2}$

$[f^{-1}(n)]' = \underline{\underline{\frac{1}{2}}}$

PROPERTIES OF EXPONENTS -

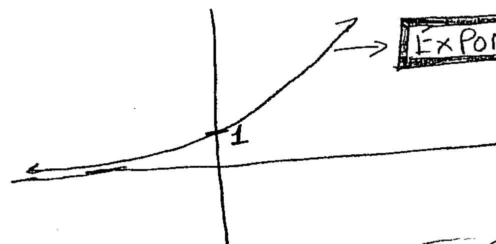
(i) $x^N x^M = x^{N+M}$

(ii) $(x^M)^N = x^{MN}$

(iii) $\frac{x^M}{x^N} = x^{M-N}$

(iv) $\left(\frac{x}{y}\right)^M = \frac{x^M}{y^M}$

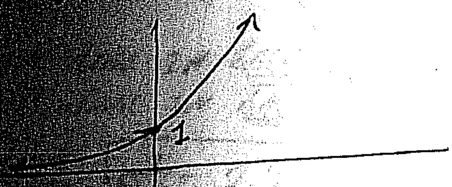
* $y = 2^x$



EXPONENTIAL CURVE

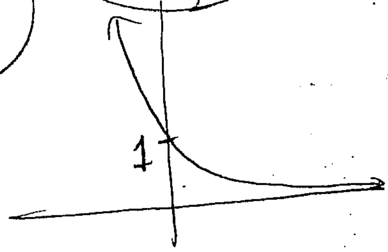
IMP

$y = 3^x$



* $y = \left(\frac{1}{2}\right)^x$

IMP

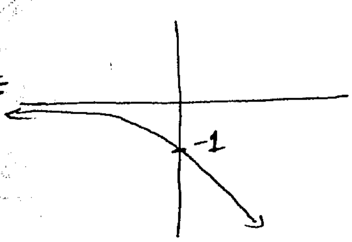


* $y = \left(-\frac{1}{2}\right)^x$ or $y = (-2)^n$

* NO CAN DO BECAUSE WE CANNOT TAKE THE SQUARE ROOT OF A NEGATIVE NUMBER.

though we can have $y = -2^n$

* $y = -2^x$



IMP

* IF $y = a^x$, & if $0 \leq a \leq 1$, THEN IT IS A DECREASING CURVE.

IF $y = a^n$, & if $a \geq 1$, THEN IT IS AN INCREASING CURVE.

* IF $y = -a^n$, THEN THE CURVE IS FLIPPED.

IMP

* EXPONENT FUNCTIONS ARE INVERSIBLE

their DOMAIN is = ALL REAL'S

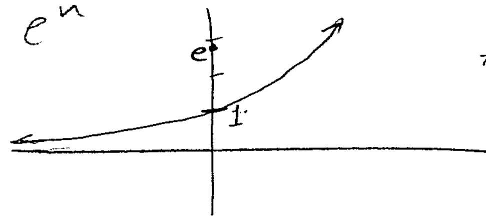
& RANGE is = ALL +IVE NUMBERS

X 9862-31483 homxb!tzathas
X 9862-36493 globe2!hkr
X 9862-39524 homxb!aramani
X 9865-26109 hogpa!pmb
X 9865-36605 hotlg!bwg
X 9867-83266 mozart!gbrill
X 9867-90548 mtqua!mask
X 9867-97825 hocus!mbr
X 9868-84131 qsun!efr
X 9868-84581 hotsb!ack
X 9868-85042 epic!limbo!mel
X 9868-88041 hocpb!ee
X 9869-21759 hogpa!music
X 9869-33050 homxb!wjs
X 9869-37780 hotld!amh
X 9869-82374 ios!brendel
X 9872-15996 torreys!adp
X 9872-32219 whamt!kl
X 9872-83733 whamt!chg
X 9875-16375 mtgzfs3!jjs
X 9875-31143 mink!jfg
X 9877-23732 honet5!csk
X 9877-25915 physics!gkc
X 9877-81808 gummo!rlh
X 9878-32381 qsun!nar
X 9878-36695 boole!mario
X 9879-13212 mhwpa!joel
X 9879-80088 alice!adele
X 9879-93105 homxb!mhwr
X 9882-21617 mtgzfs3!janef
X 9882-26899 mantic!ka2pyk
X 9882-34959 hogpa!cmg
X 9882-39769 mtgzfs3!lc
X 9882-92083 mink!dler
X 9887-87470 attme!stcjl
X 9887-94251 hogpa!ahc
X 9887-95717 holite!jez
X 9888-13744 hogpa!dchou
X 9888-13882 aloft!lai
X 9888-19073 ios!azita
X 9888-34465 hogpa!mkalk
X 9888-89428 taz!pkb1
X 9889-11144 moss!sbs
X 9889-14292 whamt!smitty
X 9889-19657 mhwpa!egs
X 9889-20350 physics!apr
X 9889-38741 mtunj!jbs2
X 9889-87408 mhwpa!mb
X 9889-94646 mantic!joem
X 9889-97342 drutx!rdn
X 9889-97761 aluxpo!jbb
X 9892-11378 qsun!dsze
X 9892-27765 hotlan!arik
X 9892-80098 hugo!ying
X 9892-80813 mtsol!fbc
X 9892-82367 vax135!gene
X 9892-83259 hound!cantor
X 9892-98571 honshu!bud
X 9897-12226 honshu!steve
X 9897-12541 hogpa!mgar
X 9897-13680 druks!escott
X 9897-14323 allwise!jpm
X 9897-39141 att!barkeep!jb
X 9897-39146 hoqaa!djk
X 9897-88502 luna!rha
X 9898-27742 aluxpo!gabi
X 9898-31852 hoqax!shah
X 9898-95437 lcuxlp!scl
X 9898-96353 hogpd!oswaldo
X 9898-97246 hrmso!andyc
X 9899-11365 mtgzfs3!iml
X 9899-12955 whamt!hal
X 9899-14245 hogpa!ppt

* $y = \pi^x$ & $y = e^x$

TMP $\boxed{e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = 2.7}$ OR $\boxed{e = \lim_{t\to 0}(1+t)^{1/t}}$

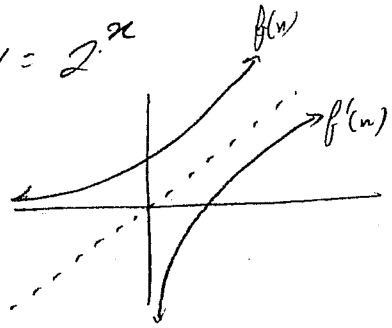
* $y = e^x$



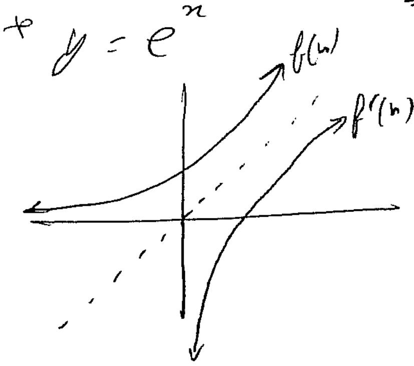
* ALSO HAS AN INVERSE

* INVERSE IS A REFLECTION ALONG LINE $y=x$ (IDENTITY LINE

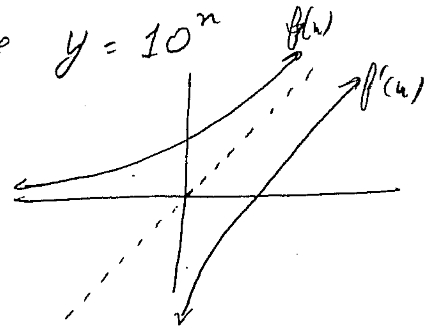
* $y = 2^x$



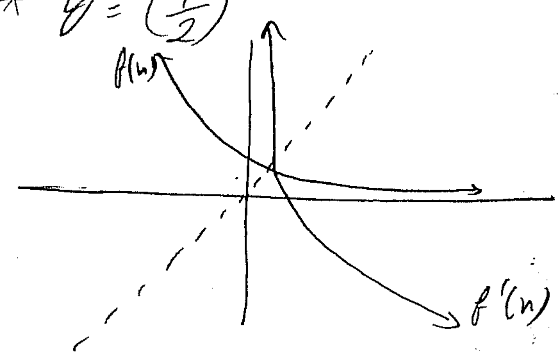
* $y = e^x$



* $y = 10^x$



* $y = \left(\frac{1}{2}\right)^x$

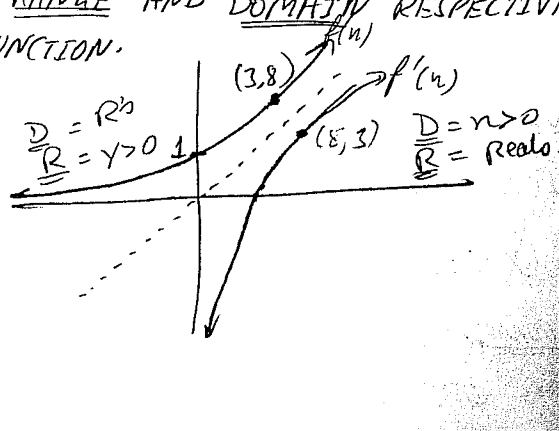


TMP * THE DOMAIN AND RANGE OF THE ORIGINAL FUNCTION RESPECTIVELY IS THE RANGE AND DOMAIN RESPECTIVELY OF THE INVERSE FUNCTION.

* $y = 2^x$

$x = 2^y$

$\Rightarrow y = \log_2 x$



* $3 = \log_2 8$

$\Rightarrow 8 = 2^3$

* $y = 10^x$

$10^y = x \Rightarrow$ Inverse

$y = \log_{10} x$

* $\log 10 = 1$

$\log 100 = 2$

$\log 1000 = 3$

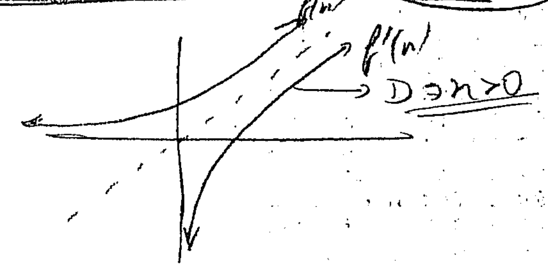
$\log 0.1 = -1$

$\log 0.01 = -2$

$\log 0.001 = -3$

IMP
- $n \to \infty \to$ Horiz. Asymptote
- $n \to \infty \to$ Area by Rectangles
- $n \to \infty \Rightarrow \lim_{n\to\infty} (1+\frac{1}{n})^n = e$

$\log -20$ = NO SOLUTION, BECAUSE LOGS ARE INVERSE FUNCTIONS AND THE DOMAIN OF LOGS DO NOT INCLUDE 0 OR NEGATIVE NUMBERS



* LOGS IN BASE 10 ARE COMMON LOGS

* LOGS IN BASE e ARE NATURAL LOGS

* $\boxed{e^1 = 2.7182818}\ldots \Rightarrow$ EULER'S #

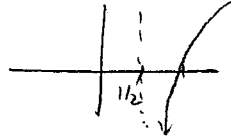
(i) $\log_a xy = \log_a x + \log_a y$ (iii) $\log_a x^t = t \log_a x$

(ii) $\log_a \frac{x}{y} = \log_a x - \log_a y$ (iv) $\log_a b = \log_{10} b / \log_{10} a$ [CHANGE OF BASE RULE] IMP

* LOG functions have VERTICAL ASYMPTODE & SO DOMAIN = V.A.

* $\log_4 (2x-1) \Rightarrow 2x-1>0$

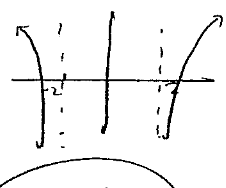
$x > \frac{1}{2}$



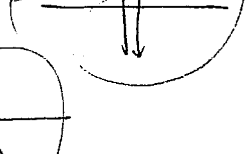
* $\log_4 (x^2-4) \Rightarrow x^2-4>0$

$x^2 > 4$

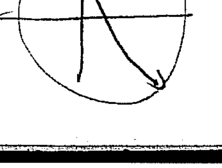
$x > \pm 2$



* A WHOLE NUMBER BASE $\Rightarrow$



* A DECIMAL BASE $\Rightarrow$



X 9829-34973 mvucl!hie
X 9829-91299 hound!alfie
X 9829-93162 hou2h!rww
X 9829-97122 mtune!hcb
X 9832-10196 hotlg!kp
X 9832-10843 homxb!david
X 9832-24312 hogpc!jcl
X 9832-84314 whamg!gjp
X 9834-11823 mrspock!raivo
X 9835-14437 hogpa!54325rt
X 9835-20124 hogpa!vjm
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X 9839-26593 gummo!fmb
X 9839-29884 lcuxlm!jma
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X 9839-37410 drutx!dmg
X 9839-82414 moss!rlm
X 9842-15108 mtqua!hyu
X 9842-16537 homxb!wlb
X 9842-22077 hrmso!wcy
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X 9844-16067 neural!patrice
X 9845-14759 drml1!russo
X 9845-21370 sos!danser
X 9845-24557 hou2h!jayh
X 9847-22683 lzsc!rac
X 9847-26801 vilya!rb
X 9847-33850 cheshire!rnh
X 9847-82431 angate!jc
X 9847-83096 att!usl!gtg
X 9847-93691 mrspock!tum
X 9847-99387 max!qhi
X 9849-10257 hogpa!hjl
X 9849-16342 arch2!rhj
X 9849-21876 hogpa!losik
X 9849-35295 whamg!ral
X 9849-35384 epic!mxc
X 9852-11842 talon1!pje
X 9852-34419 anchor!rjh
X 9852-89007 hrmso!syc
X 9852-95457 mozart!rjm
X 9855-94379 nwopb!wcjones
X 9857-15043 homxb!bc
X 9857-20936 floyd!gjn
X 9857-26008 hogpa!lurch
X 9857-34152 germany!wmt
X 9857-34784 hogpa!hhs
X 9857-98367 camille!grass
X 9858-31320 mtdcr!snc
X 9858-38629 zeppo!tyoo
X 9858-85998 hotstone!agd
X 9859-11500 aluxpo!ortner
X 9859-19485 iexist!laura
X 9859-21246 aluxpo!stretch
X 9859-25779 mtqua!wah
X 9859-29063 post!jerry
X 9859-38438 whamg!cwb
X 9862-18116 cblph!randy

GOOD EXAMPLES

* $y = e^x$

$x = e^y$

$y = \log_e x$

$\Rightarrow \boxed{y = \ln x} = \boxed{\log_e x}$

$\boxed{\ln e} \Rightarrow \boxed{e^1 = e} \Rightarrow \boxed{1}$

$\boxed{\ln 1 = \log_e 1 = 0}$

IMP

IMP

* $\log 1+x = 3$

$10^3 = 1+x$

$x = 999$

* $2^{x+4} = 8$

$\log 2^{x+4} = 2^3$

$= x = -1$

* $2^{x+4} = 7$

$\log 2^{x+4} = \log 7$

$x+4 = \frac{\log 7}{\log 2}$

$x = \frac{\log 7}{\log 2} - 4$

* $\log_2 7 = x+4$

$\frac{\log 7}{\log 2} = x+4$

* $\log x^{3/2} - \log \sqrt{x} = 5$

$\log \frac{x^{3/2}}{x^{1/2}} = 5$

$\Rightarrow \log x = 5$

$10^5 = x$

$x = 100000$

* $\ln 4x - 3\ln x^2 = \ln 2$

$\Rightarrow \ln 4x - \ln x^6 = \ln 2$

$\ln \frac{4x}{x^6} = \ln 2$

$\frac{4}{x^5} = 2$

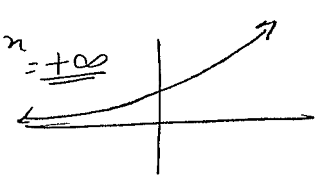
$\therefore x^5 = 2$

$x = \sqrt[5]{2}$

* $g(x) = \sqrt[4]{x}$, $f(x) = x^6$

$f \circ g(x) = (x^{1/4})^6 = \sqrt{x^3}$

* $\lim_{x \to \infty} e^x = +\infty$



THE NATURAL LOGARITHM

* $\dfrac{d \log_b x}{dx} = \lim\limits_{h \to 0} \dfrac{f(x+h) + f(x)}{h}$

$\Rightarrow \lim\limits_{h \to 0} \dfrac{\log_b (x+h) - \log x}{h}$

$\Rightarrow \lim\limits_{h \to 0} \dfrac{1}{h} \log_b (x+h) - \log x$

$\Rightarrow \lim\limits_{h \to 0} \dfrac{1}{h} \log_b \dfrac{x+h}{x}$

$\Rightarrow \lim\limits_{h \to 0} \dfrac{1}{h} \log_b \left(1 + \dfrac{h}{x}\right)$ LET $t = \dfrac{h}{x}$ $\therefore$ when $h \to 0$, th $t \to 0$

$\Rightarrow \lim\limits_{t \to 0} \dfrac{1}{tx} \log_b (1+t)$

$\Rightarrow \dfrac{1}{x} \lim\limits_{t \to 0} \dfrac{1}{t} \log_b (1+t)$

$\Rightarrow \dfrac{1}{x} \lim\limits_{t \to 0} \log_b (1+t)^{1/t}$

$\Rightarrow \dfrac{1}{x} \log_b \lim\limits_{t \to 0} (1+t)^{1/t}$ BUT $= \lim\limits_{t \to 0} (1+t)^{1/t} = \log$

$\Rightarrow \dfrac{1}{x} \log_b e$

$\Rightarrow \boxed{\dfrac{d \log_b x}{dx} = \dfrac{1}{x} \log_b e} \longrightarrow (1)$

USE (1)

* $\dfrac{d \log_e x}{dx} = \dfrac{d \ln x}{dx} = \dfrac{1}{x} \log_e e = \dfrac{1}{x} (1) = \dfrac{1}{x}$

$\therefore \boxed{\dfrac{d \ln x}{dx} = \dfrac{1}{x}, \ x > 0} \longrightarrow (2)$

* $\boxed{\dfrac{d \ln u}{dx} = \dfrac{1}{u} \dfrac{du}{dx}} \longrightarrow (3)$

* $\dfrac{d \ln |x|}{dx}$ → Here either $x > 0$ or $x < 0$. x CAN'T BE ZERO BECAUSE $\ln 0 =$ NO SOLUTION

$\dfrac{d \ln x}{dx} = \dfrac{1}{x}$; $x > 0$ & $\dfrac{d \ln -x}{dx} = \dfrac{1}{-x} (-1) = \dfrac{1}{x}$; $x < 0$

$\boxed{\dfrac{d \ln |x|}{dx} = \dfrac{1}{x}, \ x \neq 0}$

↓

(4)

```
X 9807-12001 hogpa!agd1
X 9807-23513 mtuxj!fjh
X 9807-30436 hrmso!rsr
X 9807-32878 otlan!kushal
X 9807-33505 hotlan!abbondi
X 9807-37298 physics!ht
X 9807-82404 hogpa!rick
X 9808-14823 honet6!chinnee
X 9808-81299 hrmso!ads
X 9808-98393 hotsb!rmk
X 9809-10164 hogpa!hht
X 9809-18242 hogpa!wh
X 9809-20715 physics!jdj
X 9809-24680 mhcnet!jmc
X 9809-27592 mink!gsh
X 9809-39051 gummo!fvg
X 9809-81467 homxb!mjfm
X 9809-81716 pegasus!dmt
X 9809-83176 hotld!ajc
X 9809-95497 mhcnet!abg
X 9809-99924 alux2!chester
X 9812-19884 mtqub!erl
X 9812-28195 groucho!bab
X 9812-31645 hoqub!mo
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X 9812-89428 hogpa!mms
X 9812-90167 cbosgd!drr
X 9812-91732 aluxpo!gus
X 9812-91923 mtsol!alb
X 9812-94361 hrmso!dkj
X 9812-94837 fuwutai!monin
X 9812-95413 mtgzfs3!nng
X 9815-81914 honet4!nat
X 9817-16248 iexist!slg
X 9817-18211 hogpa!aragain
X 9817-29166 pace!rgs
X 9817-84314 ulysses!rjs
X 9817-85834 hogpa!riblock
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X 9818-15243 hotsb!xue
X 9818-16053 mink!bx
X 9818-31052 hotseat!kim
X 9818-89657 ulysses!sal
X 9818-92027 globe2!csc
X 9818-92400 hogpd!hilary
X 9818-93049 mink!gyl
X 9819-18530 whamm!aps
X 9819-87683 hogpa!rds666
X 9822-25199 globe1!dlj
X 9822-30200 homxb!xxajtxx
X 9822-35129 mtnet1!matt
X 9822-35661 granjon!srh
X 9822-39744 esun!hth
X 9822-81324 granjon!bmj
X 9822-81623 hogpa!boehm
X 9822-88798 mvuxd!len
X 9822-89313 vax135!js
X 9825-98129 mtgzfs3!tpm
X 9827-18328 hou2h!davef
X 9827-31961 edsel!lbm
X 9827-93930 mtgzfs3!argyrios
X 9827-95292 physics!cgb
X 9827-97020 homxb!daf
X 9827-97486 homxb!hbz
X 9827-98828 taz!ledzep
X 9827-99421 mvuxd!vmax
X 9828-35320 whamg!knp
X 9828-35930 hoqax!chuni
X 9828-81897 whamr!jcl
X 9828-83221 hogpa!crd
X 9828-98229 mtnet1!jyu
X 9829-16727 hocpb!joee
X 9829-21266 hogpg!frg
```

* 2nd Fundamental Theorem of Calculus states that —

* if $F(x) = \int_0^x f(t)\,dt$

then $F'(x) = f(x)$

in other words

$$f(x) = \frac{d}{dx}\int_0^x f(t)\,dt$$

* $$\boxed{\ln x = \int_1^x \frac{1}{t}\,dt}$$

→ NO ABSOLUTE VALUE AS $x>0$

$$\therefore \boxed{\int \frac{1}{x}\,dx = \ln|x| + C} \rightarrow (5) \rightarrow \text{IMP}$$

* $$\frac{d\ln(x^2)}{dx} = \frac{1}{x^2}(2x) = \frac{2}{x}$$

* $$\frac{d\ln \sin x}{dx} = \frac{1}{\sin x}(\cos x) = \cot x$$

* $$\frac{d\ln \sin x^2}{dx} = \frac{1}{\sin x^2}(\cos x^2)(2x)$$

$$= 2\cot x^2$$

* $$\frac{d\ln|\sin x|}{dx} = \frac{1}{\sin x}(\cos x) = \cot x$$

* $$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$

$u = \cos x$

$du = -\sin x$

$$\Rightarrow -\int \frac{1}{u}\,du = -\int \frac{du}{u}$$

$$\Rightarrow -\ln|u| + C \Rightarrow -\ln|\cos x| + C$$

$$\int \frac{5x^4}{x^5+1}\,dx \qquad u = x^5+1, \quad du = 5x^4\,dx$$

$$= \int \frac{du}{u} \Rightarrow \ln|u| + C \Rightarrow \underline{\ln(x^5+1)+C}$$

$$\int_1^4 \frac{3}{1-2x}\,dx \qquad u = 1-2x, \quad du = -2\,dx$$

$$-\frac{3}{2}\int_1^4 \frac{du}{u} \Rightarrow -\frac{3}{2}\int_{-1}^{-7} \frac{du}{u} = -\frac{3}{2}\Big[\ln|u|\Big]_{-1}^{-7}$$

$$\Rightarrow -\frac{3}{2}\left[\ln 7 - \ln 1\right] \Rightarrow -\frac{3}{2}\left[\ln 7\right] = \underline{-\frac{3}{2}\ln 7}$$

We know that $\boxed{\log_e x = \ln x} \rightarrow (i)$ $\qquad x = 2^y$, $\log_2 x = y$

$$\therefore \boxed{\ln e = \log_e e = 1}$$

From (i) we can deduct that since

$$\boxed{\log_e x = \ln x}$$

$$\therefore \boxed{e^{\ln x} = x} \rightarrow (1)$$

IMP

$$f(x) = e^x; \quad f^{-1}(x) = \ln x$$

$$\therefore f^{-1}\circ f(x) = f^{-1}(f(x)) = f^{-1}(e^x) = \boxed{\ln e^x = \log_e e^x = x}$$

$$\therefore \boxed{\ln e^x = x} \rightarrow (2)$$

IMP

X 9788-93877 lcuxlm!dmk
X 9788-98702 pegasus!jmiros
X 9789-11514 aluxpo!fred
X 9789-14005 hogpa!pff
X 9789-22092 probe!cmv
X 9789-23048 hogpa!jn
X 9789-23628 mtqua!jmb1
X 9789-24791 hoqax!rvs
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X 9789-26896 mtunj!billb
X 9789-28337 honshu!gwg
X 9789-33309 mt747!rlm
X 9789-35852 space!sas
X 9789-37323 hrmso!jrr
X 9789-37834 research!njas
X 9789-39032 drutx!clda
X 9789-85241 moss!ljm
X 9789-95718 mtuxj!rb1
X 9790-26298 vilya!whjpk
X 9792-16808 winken!jac
X 9792-17182 hrmsf!tgc
X 9792-18064 lcuxlm!bvc
X 9792-20303 hogpa!clh
X 9792-20337 somerset!pisc2b!gene
X 9792-20768 hound!dfeit
X 9792-27506 neat!tjm
X 9792-33784 hostc6!mfp
X 9792-38351 whscad1!saa
X 9793-85016 hoh-1!koyama
X 9794-20952 mtqua!hjb
X 9794-23610 mhcnet!kwc
X 9794-29613 arch3!jaya
X 9794-35598 piccolo!ajd1
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X 9797-13991 probe!sml
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X 9797-27712 mtnet1!kcura
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X 9797-35614 hostar!nouch90
X 9797-38537 mtuxj!ljl
X 9797-81440 violin!dja
X 9797-86776 library!melia
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X 9797-93817 mtqua!swn
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X 9798-29829 mtgzfs3!rda
X 9798-35213 mantic!lla
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X 9799-25574 arch3!ilene
X 9799-25785 aluxpo!hakki
X 9799-29951 edsel!dfp
X 9802-12970 mtgzfs3!js1
X 9802-31609 hogpa!uvg
X 9802-35046 hrmso!tks
X 9802-95827 floyd!ros
X 9802-95884 hogpa!carnone
X 9805-25274 arch1!tech

LOGARITHMIC DIFFERENTIATION V.V.IMP

* $y = \frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}$ → (1) → Cannot use properties of logs

* $y = \ln\left(\frac{x^2\sin x}{\sqrt{1+x}}\right)$ → (2) → Use the properties of logs

(2) ⇒ $\Rightarrow d\left[\ln x^2\sin x - \ln\sqrt{1+x}\right]$

$\Rightarrow d \ln x^2 + \ln\sin x - \frac{1}{2}\ln(1+x)$

$\Rightarrow d\left[2\ln x + \ln\sin x - \frac{1}{2}\ln(1+x)\right]$

$\Rightarrow \frac{2}{x} + \frac{1}{\sin x}(\cos x) - \frac{1}{2}\left(\frac{1}{1+x}\right)(1)$

$\Rightarrow \frac{2}{x} + \frac{\cos x}{\sin x} - \frac{1}{2+2x} \Rightarrow \frac{2}{x} + \cot x - \frac{1}{2+2x}$

(1) ⇒ ln both sides of the equation -

$\ln y = \ln \frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}$

⇒ Solving implicitly, we get →

$\frac{1}{y}\frac{dy}{dx} = 2\ln x + \frac{1}{3}\ln(7x-14) - 4\ln(1+x^2)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{7}{3(7x-14)} - 4\frac{1}{(1+x^2)}(2x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{7}{3(7x-14)} - \frac{8x}{1+x^2}$

multiply both sides by 'y'

$\frac{dy}{dx} = y\left(\frac{2}{x} + \frac{7}{21x-42} - \frac{8x}{1+x^2}\right)$

Substitute for y

$\frac{dy}{dx} = \frac{x^2\sqrt[3]{7x-14}}{(1+x^2)^4}\left(\frac{2}{x} + \frac{7}{21x-42} - \frac{8x}{1+x^2}\right)$

$\boxed{\frac{de^x}{dx} = e^x}$ IMP $\boxed{\frac{de^u}{dx} = e^u \frac{du}{dx}}$ IMP

$\boxed{\int e^x dx = e^x + c}$

* $y = e^{\sin x} = e^{\sin x}(\cos x)$
* $y = e^{x^2+2x} = e^{x^2+2x}(2x+2)$
* $\int \cos x \, e^{\sin x} dx =$ $\quad u = \sin x$, $du = \cos x\,dx$

$\int e^u du \Rightarrow e^{\sin x} + c$

* $\int 2x\, e^{x^2} dx \Rightarrow$ $\quad u = x^2$, $du = 2x\,dx$

$\int e^u du = e^{x^2} + c$

* $y = a^x$ (a = constant)

(i) $\frac{da^x}{dx}$, (ii) $\int a^x dx$

If $y = a^x$, then

$\ln y = \ln a^x$

$\ln y = x \ln a$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x(0)\ \ln a(1) \leftarrow \boxed{\frac{d \ln a}{dx} = 0}$ -IMP $\quad \log_e a =$ IMP

$\Rightarrow \frac{dy}{dx} = y \ln a$

$\boxed{\frac{da^x}{dx} = a^x \ln a}$ or $\boxed{\frac{da^u}{dx} = a^u \ln a \left(\frac{du}{dx}\right)}$ IMP

X 9777-80262 mt747!chm
X 9777-94281 hogpf!wcm
X 9777-96239 ho1focus!dko
X 9777-97599 alice!rvc
X 9777-98304 hogpa!whlm
X 9777-98460 angate!jsm
X 9777-99457 whfss1!mhh
X 9778-20381 hotstone!rjr
X 9778-28419 mtunm!ricci
X 9778-30242 bubba!avs
X 9778-97666 hotsun!mbn
X 9779-19215 homxa!russw
X 9779-21562 mtsol!then
X 9779-22251 hogpa!dgd
X 9779-23659 qsun!rvh
X 9779-24035 honet4!mchen1
X 9779-27851 hou2h!shelley
X 9779-28235 boole!clark
X 9779-29438 hogpa!hrl
X 9779-81086 hotlg!jrc
X 9779-83801 homxb!htb
X 9779-86908 hound!rule
X 9782-12185 hrcms!may
X 9782-12668 groucho!kmd
X 9782-15132 homxb!fkovacs
X 9782-16628 allegra!amg
X 9782-25845 fmg4a!pete
X 9782-27218 vax135!dcg
X 9782-28982 mvuxb!rael
X 9782-30888 arch3!alanw
X 9782-31061 mtgzfs3!mcohen
X 9782-34188 edsel!melici
X 9782-34839 hotlg!ks
X 9782-34944 ios!ar
X 9782-37638 mtqua!ng
X 9782-81338 mtdcr!rob
X 9782-88260 aluxpo!musc
X 9782-88469 hrmsf!sam
X 9782-93304 houxa!gisele
X 9782-98760 lzsc!slr
X 9783-90218 corona!cameron
X 9784-80307 hrmso!am
X 9784-83428 spin!dabm
X 9784-89866 moss!psg
X 9784-92427 hugo!jms
X 9784-93267 hogpa!wsc
X 9785-18115 honet6!k2pbt
X 9785-18567 allwise!lfmatt
X 9785-32416 hotlg!amw
X 9786-92235 boole!jj
X 9786-97994 wmsa!aks
X 9786-99358 wmsa!anilm
X 9786-99973 qsun!galia
X 9787-14874 esun!jbs
X 9787-21726 aloft!mbs
X 9787-22497 qsun!lkkwa
X 9787-31032 hoqax!gtt
X 9787-39205 molson!jmh
X 9787-82251 hoh-1!pjf
X 9787-84023 allegra!btm
X 9787-84482 mtung!tom1
X 9787-88061 mtuxo!lge
X 9787-91679 granjon!gmg
X 9787-96840 wisny!rdy
X 9788-21257 hrmso!mky
X 9788-24823 mtnet1!greg
X 9788-25108 trumpet!rjg
X 9788-26879 houxa!mbn
X 9788-32101 whamr!ckr
X 9788-32319 mtnet1!kk
X 9788-36402 research!prem
X 9788-37684 hos1cad!jc1
X 9788-82777 mrspock!lwc

$\frac{d2^x}{dx} = 2^x \ln 2$ * $\frac{d\,2^{\sin x}}{dx} = 2^{\sin x} \ln 2 (\cos x)$

* $\frac{d\pi^x}{dx} = \pi^x \ln \pi$

* $\frac{d\,4^{2x^2+x}}{dx} = 4^{2x^2+x} \ln 4 (4x+1)$

* $\frac{da^x}{dx} = a^x \ln a$

$\Rightarrow \frac{1}{\ln a} \frac{da^x}{dx} = a^x$

$\therefore \boxed{\int a^x dx = \frac{a^x}{\ln a}}$

* $\int 4^x dx = \frac{4^x}{\ln 4} + C$ * $\int \pi^x dx = \frac{\pi^x}{\ln \pi} + C$

* $\int 3^x dx = \frac{3^x}{\ln 3} + C$ * $\int x^2 4^{x^3} dx =$ $u = x^3$, $du = 3x^2$

* $\int \cos x\, e^{\sin x} dx$ $\Rightarrow \frac{1}{3} \int 4^u du$

$\int e^u du$ $u = \sin x$, $du = \cos x\, dx$ $\Rightarrow \frac{1}{3} \frac{4^u}{\ln 4} + C$

$\frac{e^u}{\ln e} + C$

$\Rightarrow \frac{e^{\sin x}}{\ln e} + C$

$$\frac{d e^{4x^2+2x}}{dx} = e^{4x^2+2x}(8x+2)$$

$$\frac{d\, 2^{4x^2-2x}}{dx}$$

$$\Rightarrow 2^{4x^2-2x} \cdot \ln 2\,(8x+2)$$

$$y = x^x$$

$$\frac{dx^x}{dx} = y = x^x$$

$$\Rightarrow \ln y = \ln x^x$$

$$\frac{1}{y}\frac{dy}{dx} = x \ln x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x(1)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \ln x$$

$$\therefore \frac{dy}{dx} = y(1+\ln x)$$

$$\Rightarrow \boxed{\frac{dx^x}{dx} = x^x(1+\ln x)}$$

$$\frac{de}{dx} = 0, \quad \frac{de^2}{dx} = 0, \quad \frac{de^3}{dx} = 0, \quad \frac{de^x}{dx} = e^x$$

$$\frac{d2^x}{dx} = 2^x \ln 2, \quad \frac{d\pi^x}{dx} = \pi^x \ln \pi,$$

$$\ln(x+y) = x$$

$$e^x = x+y$$

$$y = e^x - x$$

$$\frac{dy}{dx} = e^x - 1$$

$$\frac{de^{-x}}{dx} = e^{-x}(-1) = -e^{-x}$$

$$\int_0^2 \frac{x^2-2}{x+1}\,dx$$

$$x+1 \overline{)\,x^2 - 2\,}\;(x-1$$

$$x^2 + x$$

$$-x-2$$

$$-x-1$$

$$-1$$

$$\Rightarrow \int_0^2 x - 1 + \left(\frac{-1}{x+1}\right)$$

X 9702-29907 whamt!jna3
X 9702-33414 homxb!jcx
X 9702-36290 mink!guth
X 9702-38189 hlwpi!jwasielewski
X 9702-80144 hogpa!duff
X 9702-80669 whamr!paul
X 9702-83967 ulysses!bjs
X 9702-84240 hocpa!mchin
X 9702-88492 kato!bacc
X 9702-88859 mtqua!ams
X 9702-88924 aluxpo!jfl
X 9702-89045 hogpa!kurek
X 9702-90043 mtunm!wbs
X 9702-90303 mhuxo!vh
X 9702-95146 gummo!rke
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X 9714-11083 mhcnet!tct
X 9714-14289 mtunh!tps
X 9714-17998 ulysses!vss
X 9714-37511 aloft!jkim
X 9714-96869 mink!grewal

SEPARABLE DIFFERENTIAL EQUATIONS

Solve $\frac{dy}{dx} = 3xy \rightarrow$ Has a "y" $\therefore$ cannot Integrate.

but we can solve $\frac{dy}{dx} = 3x$

$\therefore dy = 3x\,dx$

Integrate both sides

$\int dy = \int 3x\,dx$

$\therefore y + C = \frac{3x^2}{2} + C$

$\therefore y = \frac{3x^2}{2} + C$

IMP

But — $\frac{dy}{dx} = 3xy$

$dy = 3xy\,dx$

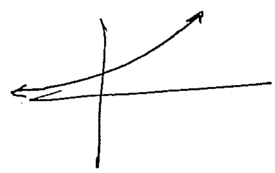
$\frac{dy}{y} = 3x\,dx$

$\int \frac{dy}{y} = \int 3x\,dx$

$\ln|y| = \frac{3x^2}{2} + C$

$e^{\frac{3x^2}{2} + C} = |y|$

↳ IS ALWAYS POSITIVE



e to any number is +ive

$|y| = e^{\frac{3x^2}{2}} \cdot e^{c}$

But $e^c = K$ (constant)

$\therefore |y| = K e^{3x^2/2}$

* $\frac{dy}{dx} = \frac{x}{y^2}$

$y^2\,dy = x\,dx$

$\int y^2\,dy = \int x\,dx$

$\therefore \frac{y^3}{3} = \frac{x^2}{2} + C$

$$y^3 = \frac{3x^2}{2} + c$$

$$y = \sqrt[3]{\frac{3x^2}{2} + c}$$

$$x(y-1)\frac{dy}{dx} = y$$

$$\frac{y-1}{y}\,dy = \frac{dx}{x}$$

$$\int \frac{y-1}{y}\,dy = \int \frac{dx}{x} \Rightarrow \int 1 - \frac{1}{y}\,dy = \int \frac{dx}{x}$$

$$\Rightarrow y - \ln|y| = \ln|x| + c$$

$$\Rightarrow y = \ln|x| + \ln|y| + c$$

$$\therefore y = \ln|xy| + c$$

SEPARABLE INITIAL VALUE DIFFERENTIAL EQUATION

* $\frac{dy}{dx} = -4xy^2 \quad y(0) = 1$

$$\frac{dy}{y^2} = -4x\,dx$$

$$\int \frac{dy}{y^2} = \int -4x\,dx$$

$$\frac{y^{-1}}{-1} = \frac{-4x^2}{2} + c \rightarrow (1)$$

But $y(0)$ when $x = 0$, then $y(0) = 1$

$$\frac{1}{-1} = -2(0)^2 + c \Rightarrow c = -1$$

Substitute c in (1)

$$\Rightarrow \frac{-1}{y} = -2x^2 - 1$$

$$-1 = y(-2x^2 - 1) \Rightarrow y = \frac{-1}{-2x^2 - 1}$$

... Growth Notes

A quantity is said to have an exponential growth (decay) if at each instant of time its rate of increase (decrease) is proportional to the amount of the quantity present.

... natural quantities change in time at an instantaneous rate ...ending on the value of the quantity itself. For example, the rate ... which the amount of radioactive substance decreases because ... radioactive desintegration depends on the amount of sustance present. Also - the rate at which money in a saving account at compound interest increases depends on the amount of money in the account.

...ally, if x represents the quantity of some substance at time t, then $\frac{dx}{dt}$ represents the instantaneous rate of change of x. If $\frac{dx}{dt}$ is proportional to the quantity x then we have $\frac{dx}{dt} = kx$ where k is the constant of the proportion. (Sometimes called growth rate)

K = rate of change.

If $K>0$, then $\frac{dx}{dt}>0$ so x is increasing as time goes on

$k<0$, then $\frac{dx}{dt}<0$ so x is decreasing as time goes on.

We can solve this differential equation as follows.

$\frac{dx}{dt} = kx$

$\frac{dx}{x} = k\,dt$

$\int\frac{dx}{x} = \int k\,dt$

$\ln|x| = kt + C$

$e^{\ln|x|} = e^{kt+C}$

$|x| = e^{kt}\cdot e^{C}$

since $|x| = \pm x$

$x = (\pm e^{C})(e^{kt})$

if we let x_0 be the value of x when $t=0$

then

$X_0 = \pm e^{C}\cdot e^{k\cdot 0} = \pm e^{C}$

so $\boxed{x = x_0 e^{kt}}$

Example 1: A culture of bacteria grows so that the rate of change of the population is proportional to the population. Suppose the population was originally 100 and 2 days later it is 100,000. How many days does it take for the population to reach 10^{11}. this is a $\frac{dx}{dt} = kx$ problem

$X = X_0 e^{kt}$; $X_0 = 100$

$t = 2$, $x = 100{,}000$

what is t when $x = 10^{11}$

$\boxed{\ln 10^{3/2} = K}$

$X = 100e^{kt}$

$100{,}000 = 100e^{2k}$

$1000 = e^{2k}$

$\ln 1000 = 2k$

$\frac{1}{2}\ln 10^3 = k$ Find K first.

$\therefore X = 100e^{(\ln 10^{3/2})t}$

$X = 100e^{\ln 10^{3/2 t}}$

$X = 100\cdot 10^{3/2 t}$

$10^2\cdot 10^{3/2 t} = 10^{3/2 t + 2}$

if $x = 10^{11}$

$10^{11} = 10^{3/2 t + 2}$

$t = 6$ Ans 6 days

$f \circ f^{-1}(x) = f(f^{-1}(x)) = x$

$\frac{dy}{dx}$ of Inverse $= 1/dx/dy$

3) $e = \lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^{n} = 2.7$ or $e = \lim_{t \to 0}(1+t)^{1/t}$

4) $\log_a b = \log_{10} b / \log_{10} a$ → change of base rule.

5) $\log_e x = \ln x$

6) $\ln e = \log_e e = 1$

7) $\ln 1 = \log_e 1 = 0$

8) $\frac{d \log_b x}{dx} = \frac{1}{x} \log_b e$

9) $\frac{d \ln x}{dx} = \frac{1}{x}, \ x > 0$ $x \neq 0$

10) $\frac{d \ln u}{dx} = \frac{1}{u} \frac{du}{dx}; \ u > 0$

11) $\frac{d \ln |x|}{dx} = \frac{1}{x}, \ x \neq 0$

12) $\ln x = \int_1^x \frac{1}{t}\, dt$ → Second Fundamental theorem of Calculus.

12) $\int \frac{1}{x}\, dx = \ln|x| + C$

14) $\log_e x = \ln x \therefore e^{\ln x} = x$

15) $\ln e^x = \log_e e^x = x$ $\frac{d \ln e^x}{dx} = \frac{dx}{dx} = 1$

16) $\frac{d e^x}{dx} = e^x$ or $\frac{d e^u}{dx} = e^u \frac{du}{dx}$

17) $\int e^x\, dx = e^x + C$

18) $\boxed{\frac{d \ln a}{dx} = 0}$

```
Apparently-To: hotlg!ks
Apparently-To: hotlg!bbs
Apparently-To: hotlg!bala
Apparently-To: hotld!umm
Apparently-To: hotld!rrp
Apparently-To: hotld!ram1
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Apparently-To: hostar!kpd
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Apparently-To: hoqub!hoibpa!nar
Apparently-To: hoqub!ada
Apparently-To: hoqub!acg
Apparently-To: hoqax!shah
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Apparently-To: hoqaa!pramila
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Apparently-To: honshu!lalita
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Apparently-To: honet5!arao
Apparently-To: honet5!amit
Apparently-To: honet4!vrk
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Apparently-To: honet4!bhusri
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Apparently-To: homxc!vax135!aravind
Apparently-To: homxc!trr
Apparently-To: homxc!tap
Apparently-To: homxc!swam
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Apparently-To: homxc!sekar
Apparently-To: homxc!seema
Apparently-To: homxc!sc
Apparently-To: homxc!rnp1
Apparently-To: homxc!reddy
Apparently-To: homxc!razdan
```

19) $\frac{da^x}{dx} = a^x \ln a$; $\frac{da^u}{dx} = a^u \ln a \frac{du}{dx}$

20) $\int a^x dx = \frac{a^x}{\ln a}$

21) $\frac{dx^x}{dx} = x^x(1+\ln x)$

22) $\frac{de}{dx} = 0$, $\frac{de^2}{dx} = 0$, $\frac{de^3}{dx} = 0$, $\frac{de^x}{dx} = e^x$

23) $\frac{d2^x}{dx} = 2^x \ln 2$

24) $\frac{d5^x}{dx} = 5^x \ln 5$, $\frac{de^{-x}}{dx} = e^{-x}(-1) = -e^{-x}$

25) $\lim_{x \to +\infty} e^x = +\infty$

26) $\lim_{x \to -\infty} e^x = 0$

27) $\lim_{x \to +\infty} \ln x = +\infty$

28) $\lim_{x \to 0^+} \ln x = -\infty$

29) $\lim_{x \to +\infty} e^{-x} = \lim_{x \to +\infty} \frac{1}{e^x} = 0$

30) $\lim_{x \to -\infty} e^{-x} = \lim_{x \to -\infty} \frac{1}{e^x} = +\infty$

31) $\lim_{x \to +\infty} \frac{e^x}{x^N} = +\infty$

32) $\lim_{x \to +\infty} \frac{\ln x}{x^N} = 0$

33) $\lim_{x \to +\infty} \frac{x^N}{e^x} = 0$

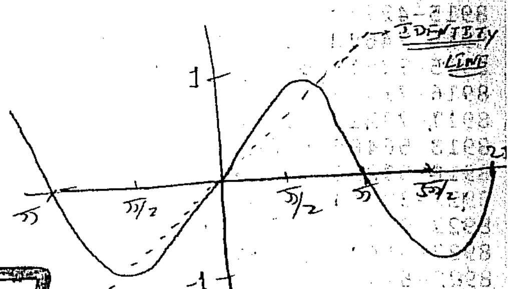
34) $\lim_{x \to +\infty} \frac{x^N}{\ln x} = +\infty$

8. INVERSE TRIGONOMETRIC FUNCTIONS

The basic Trigonometric functions are called Periodic functions.
Periodic functions are not one to one and so they do not pass the horizontal line test, therefore they have no inverses.
The $\sin x = y$ curve is defined on the entire interval $(-\infty, +\infty)$
We can find the Inverse of a periodic function by limiting its domain

$\sin^{-1}(\sin y) = y$ if $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

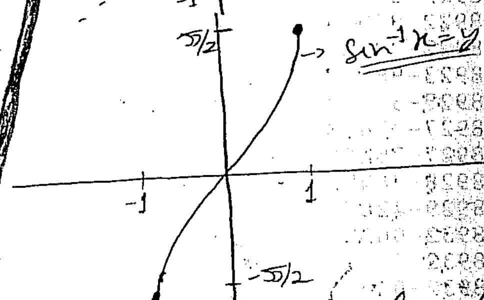
$\sin(\sin^{-1} x) = x$ if $-1 \le x \le 1$



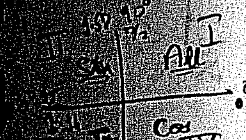
If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then

$y = \sin^{-1} x$ and $x = \sin y$

are equivalent statements.

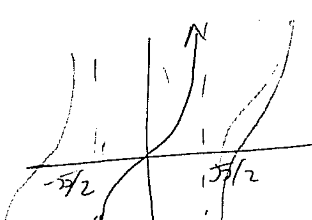


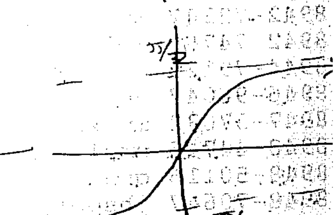
The Domain of the original function is the Range of the inverse function and the Domain of the inverse function is the Range of the original function



* Arcsin is defined only in quadrants I & IV and is given as a NEGATIVE number in the 4th quadrant.

Inverse tangent function





$\tan^{-1}(\tan y) = y$, if $-\frac{\pi}{2} < y < \pi/2$

$\tan(\tan^{-1} x) = x$, if $-\infty < x < +\infty$

If $-\infty < x < +\infty$ and $-\pi/2 < y < \pi/2$, then
$y = \tan^{-1} x$ and $\tan y = x$
are equivalent statements.

* Arctan is defined only in the quadrants I & IV

X 8904-87893 hound!bwp
X 8907-32301 research!dep
X 8908-17986 hogpa!taoyuan
X 8908-58893 boole!sanjay
X 8908-58901 allegra!jag
X 8908-97104 research!ken
X 8908-98505 mtgzfs3!warner
X 8910-57096 mtqua!mdl
X 8911-85838 whamg!wpyung
X 8914-00609 mink!ims
X 8914-12660 hogpa!cjb
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X 8962-29204 ihwpg!mo
X 8962-62596 aluxpo!ccl
X 8962-64838 worf!padma
X 8963-38167 iexist!tschirgi

* Inverse Secant functions

$\sec^{-1}(\sec y) = y$ if $0 \le y < \pi/2$ or $\pi \le y < 3\pi/2$

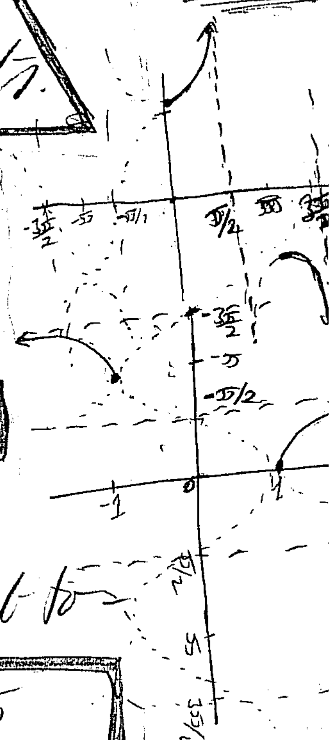
$\sec(\sec^{-1} x) = x$ if $x \le -1$ or $x \ge 1$

* If $|x| \ge 1$ and if $0 \le y < \pi/2$ or $\pi \le y < 3\pi/2$, then

$y = \sec^{-1} x$ & $\sec y = x$

are equivalent statements.

quad I & III



* $y = \cos^{-1} x$ is equivalent to

$x = \cos y$ if $0 \le y \le \pi$

$-1 \le x \le 1$

quad I & II

* $y = \cot^{-1} x$ is equivalent to

$x = \cot y$ if $0 < y < \pi$

$-\infty < x < +\infty$

quad I & II

* $y = \csc^{-1} x$ is equivalent to

$x = \csc y$ if $0 < y \le \pi/2$

or

$-\pi < y \le -\pi/2$

quad I & III

$|x| \ge 1$

$$\boxed{\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}}$$

$$\boxed{\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}}$$

$$\boxed{\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}}$$

PRACTICE EXERCISE 8.1 IN THE BOOK &

EXE. 8.2

$$\boxed{\sin^{-1}(-x) = -\sin^{-1}x} \to \text{ODD}$$

$$\boxed{\tan^{-1}(-x) = -\tan^{-1}x} \to \text{ODD}$$

$$\boxed{\sec^{-1}(-x) = \pi + \sec^{-1}x \quad \text{if } x \geq 1} \to \text{EVEN}$$

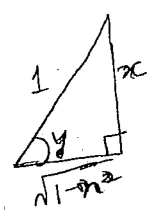
DERIVATIVES & INTEGRALS INVOLVING INVERSE TRIG. FUNCTIONS

* $y = \sin^{-1}x \Rightarrow$ function $\quad \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

$\Rightarrow x = \sin y$

$\frac{dx}{dy} = \cos y \frac{dy}{dy}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$ In terms of y



$\therefore \sin \cdot \cos y = \sqrt{1-x^2}$

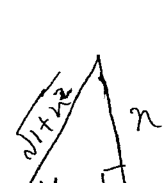
$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

* $y = \tan^{-1}x$

$x = \tan y$

$\frac{dx}{dy} = \sec^2 y \frac{dy}{dy}$

$= \frac{dy}{dx} = \frac{1}{\sec^2 y}$



$\therefore \sec y = \sqrt{1+x^2}$

x 8848-22036 research!jon
x 8848-33044 torreys!rmm
x 8848-98058 violin!bam
x 8850-66617 honet4!sbose
x 8851-86344 violin!jmlin
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x 8852-29110 longs!rab
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x 8900-68110 hogpb!miyaji
x 8902-05899 aluxpo!akd
x 8902-29621 ihlpl!nagds
x 8902-69741 honet9!chow
x 8903-57130 aloft!share! aijeet
x 8903-91722 lcuxlm!rtr
x 8904-27131 gummo!arizon
x 8904-39041 physics!jo

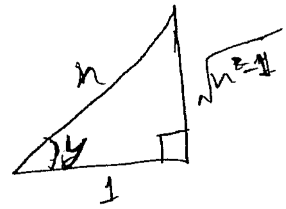
$$= \frac{d \tan^{-1} n}{dn} = \frac{1}{1+n^2}$$

* $y = \sec^{-1} n$

$n = \sec y$

$$\frac{dn}{dy} = \sec y \tan y \frac{dy}{dy}$$

$$\frac{dy}{dn} = \frac{1}{\sec y \tan y}$$



$$\frac{dy}{dn} = \frac{1}{n(\sqrt{n^2-1})}$$

1) $\frac{d[\sin^{-1} n]}{dn} = \frac{1}{\sqrt{1-n^2}}$

2) $\frac{d[\cos^{-1} n]}{dn} = -\frac{1}{\sqrt{1-n^2}}$

3) $\frac{d[\tan^{-1} n]}{dn} = \frac{1}{1+n^2}$

4) $\frac{d[\cot^{-1} n]}{dn} = -\frac{1}{1+n^2}$

5) $\frac{d[\sec^{-1} n]}{dn} = \frac{1}{n\sqrt{n^2-1}}$

6) $\frac{d[\csc^{-1} n]}{dn} = -\frac{1}{n\sqrt{n^2-1}}$

MOST IMP

1) $\int \frac{dn}{\sqrt{a^2-n^2}} = \sin^{-1}\frac{n}{a} + C$

2) $\int \frac{-dn}{\sqrt{a^2-n^2}} = -\int \frac{dn}{\sqrt{a^2-n^2}}$

3) $\int \frac{dn}{a^2+n^2} = \frac{1}{a}\tan^{-1}\frac{n}{a} + C$

4) $\int \frac{-dn}{a^2+n^2} = -\int \frac{dn}{a^2+n^2}$

5) $\int \frac{dn}{n\sqrt{n^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{n}{a} + C$

6) $\int \frac{-dn}{n\sqrt{n^2-a^2}} = -\int \frac{dn}{n\sqrt{n^2-a^2}}$

MOST IMP

5.

X 8766-95912 honet4!vrk
X 8767-17326 hrojr!rrr
X 8771-16338 blanca!cinj
X 8771-30589 hoqax!ecaby
X 8772-08475 homxc!ggw
X 8772-90844 nwscb!hal
X 8774-22360 houxa!em
X 8774-51575 hocus!enrique
X 8776-97591 torreys!scw
X 8777-30999 abars!srn
X 8778-73410 alice!ys
X 8780-75632 drutx!hearn
X 8782-99872 research!lhn
X 8786-26819 drutx!nks
X 8787-58655 physics!wkang
X 8787-81475 hogpa!4383rle
X 8789-00060 aluxpo!ykp
X 8789-44343 karin!pht
X 8791-24904 drutx!grb
X 8792-08788 camille!ruth
X 8792-17582 hoh-1!mvs
X 8792-18421 attme!orwjf
X 8793-34395 hoibpa!jsl
X 8794-69254 mt747!jskuo
X 8794-69784 mhcnet!fcc
X 8795-06966 att!gracey!jchang
X 8796-34627 drutx!roller
X 8797-52790 germany!mlx
X 8798-61351 blink!nikos
X 8799-67557 research!mkearns
X 8802-80656 ihlpf!vbe
X 8803-93679 mvucl!chc
X 8811-81278 ulysses!herman
X 8812-25171 drutx!rgk
X 8812-33097 torreys!raj
X 8812-84226 lzsc!tcc
X 8812-86727 homxb!flint77
X 8812-96916 druwa!rxr
X 8814-32275 iwtin!rdp
X 8817-17176 physics!dvl
X 8817-19519 torreys!jdw
X 8817-24900 iwcs!iwtpm!tjc
X 8817-29316 iwtil!djtdjt
X 8818-19150 woomera!jhe
X 8818-32464 ihlpf!jsp
X 8818-86666 hogpa!rathman
X 8822-16411 drutx!ars
X 8822-80461 hoqax!pgb
X 8823-31525 cbvox!ashraf
X 8823-34371 ihspa!babu
X 8825-26198 torreys!tem
X 8826-11208 ihlpl!jpmc
X 8826-33059 nwscb!murf
X 8826-33128 druwa!ljg
X 8827-15889 mvuxd!lr
X 8829-14886 iexist!jloos
X 8831-86901 homxa!science
X 8832-19675 torreys!jej
X 8832-23147 drutx!jrl
X 8832-33153 iexist!gaylord
X 8833-16863 cbvox!clyde
X 8834-17240 aluxpo!caf
X 8834-35211 edsel!dcb
X 8834-80972 ihspc!rap
X 8836-91419 iexist!djcarr
X 8838-13626 mtgzfs3!ekg
X 8838-91535 drutx!lpp
X 8842-24288 hoqaa!jls
X 8842-97728 ulysses!erg
X 8844-85116 barkeep!bude
X 8844-88996 hogpa!eshwar
X 8845-23499 mtnms!jlc
X 8848-12399 druwa!jpm

A
1) $\sin^2\theta + \cos^2\theta = 1$
2) $1 + \tan^2\theta = \sec^2\theta$
3) $\cot^2\theta + 1 = \csc^2\theta$

B
1) $\sin(-t) = -\sin t$
2) $\cos(-t) = +\cos t$
3) $\tan(-t) = -\tan t$
4) $\csc(-t) = -\csc t$
5) $\sec(-t) = +\sec t$
6) $\cot(-t) = -\cot t$

C
1) $\cos 2t = \cos^2 t - \sin^2 t$
or
$1 - 2\sin^2 t$
or
$2\cos^2 t - 1$

2) $\sin 2t = 2\sin t\cos t$

3) $\tan 2t = \dfrac{2\tan t}{1-\tan^2 t}$

D)
1) $\cos\frac{1}{2}t = \pm\sqrt{\dfrac{1+\cos t}{2}}$
2) $\sin\frac{1}{2}t = \pm\sqrt{\dfrac{1-\cos t}{2}}$
3) $\tan\frac{1}{2}t = \pm\sqrt{\dfrac{1-\cos t}{1+\cos t}}$

E
1) $\cos(t+s) = \cos t\cos s - \sin t\sin s$
$\cos(t-s) = \cos t\cos s + \sin t\sin s$

2) $\sin(t+s) = \sin t\cos s + \sin s\cos t$
$\sin(t-s) = \sin t\cos s - \sin s\cos t$

3) $\tan(t+s) = \dfrac{\tan t + \tan s}{1 - \tan t\tan s}$

$\tan(t-s) = \dfrac{\tan t - \tan s}{1 + \tan t\tan s}$

General form –

$\boxed{F(x) = -a\sin(kx+b) \pm c}$ → IMP

1) $\boxed{|a|}$ determines the amplitude or Range

(i) If $a>1$, vertical stretch

(ii) If $0<a<1$, vertical shrink

(iii) If $a<0$, Flip the curve

* Fund. Period 0 to 2π Sin & Cos curve

2) 'k' changes the period of the curve – $\frac{2\pi}{|k|}$ → IMP

3) $\boxed{0 \le kx+b < 2\pi}$ → gives the phase shift

where the curve begins

→ where the curve ends.

4) Addition or subtraction of a constant moves the curve up or down.

* Fund. Per of tan curve = 0 to π

phase shift = $\boxed{-\frac{\pi}{2} < kx+b < \frac{\pi}{2}}$ → IMP

Period = $\boxed{\frac{\pi}{|k|}}$

2a) Supp. Pg 507

X	8597-32056	research!wow
X	8597-34199	drutx!jfh
X	8598-19022	mtnet1!srh
X	8598-20714	drutx!sfj
X	8598-38725	torreys!drb
X	8598-82591	hrmso!jkh
X	8598-96818	hrmso!ckc
X	8610-95530	cbvox!pjb
X	8615-88281	clockwise!lyoung
X	8615-89415	mtqua!gr
X	8619-12561	houxa!ngh
X	8625-86841	cbnmva!scc
X	8629-30726	hotlan!mouli
X	8629-36071	aluxpo!yso
X	8641-08976	hotlg!dmz
X	8660-38682	probe!syy
X	8661-41156	hound!guy9
X	8665-38646	hou2h!fwade
X	8680-13871	ios!cyy
X	8680-90429	attme!ortsf
X	8681-05759	honet4!us
X	8689-22314	whamr!c5588
X	8702-16332	neptune!bsmith
X	8702-82478	mtgzfs3!ktf
X	8702-86815	ihspa!shs
X	8707-13980	hogpa!net
X	8707-30782	houxa!sko
X	8707-80392	whamg!twc
X	8712-24365	zeppo!miles
X	8712-88226	globe2!sxc
X	8713-10817	ihspa!dklein
X	8714-10250	mhcnet!clh
X	8714-10252	whamt!jjcc
X	8714-16356	hogpa!shamst
X	8716-91245	wisny!yeh
X	8718-16031	camille!maryl
X	8718-31187	lcuxlm!et
X	8724-14403	druwa!vader!das
X	8724-34129	physics!msh
X	8725-83308	attsb!gid
X	8726-14233	mtunh!jpy
X	8726-30239	longs!minh
X	8726-32801	hotsc!jwr
X	8726-99560	ihlpf!carper
X	8727-24360	lccras!dwv
X	8727-85754	qsun!bobs
X	8732-36101	hotlg!ash
X	8732-48312	hoibpa!kamvar
X	8733-22134	ulysses!sgl
X	8734-81731	mvuxd!gt
X	8734-84391	mvuas!jcc
X	8734-89271	homxb!kchin
X	8735-02452	mtung!dls
X	8737-44258	hoserve!bes
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X	8744-90576	drutx!hi
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X	8747-80268	hocpb!haliski
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X	8748-38923	research!tom
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X	8758-39501	research!dbm
X	8758-61297	likewise!cpc
X	8758-64260	mhuhk!pai
X	8759-07137	whservd!ramesh
X	8759-08796	whamt!ysp
X	8762-84643	cbnea!liz

$\int u\,dv = u\cdot v - \int v\,du$ LIPET

$\int \ln x\,dx = x\ln x - x + C$

$(f^{-1})'(x) = \frac{1}{f'[f^{-1}(x)]}$

MOST IMP

L'HOSPITAL'S RULE → SEE DITTO.

TRAPEZOIDAL RULE

* $\int xe^x dx$ & $\int e^x \cos x\, dx$

INTEGRATION BY PARTS

* $\int \frac{d}{dx} f(x) \cdot g(x) = \int f(x) \cdot g'(x) + g(x) \cdot f'(x) dx$

$\Rightarrow f(x) \cdot g(x) + C = \int f(x) \cdot g'(x) + \int g(x) \cdot f'(x) dx$

$\Rightarrow f(x) g(x) - \int g(x) \cdot f'(x) = \int f(x) g'(x)$

$u = f(x)$
$v = g(x)$
$du = f'(x) dx$
$dv = g'(x) dx$

$= \boxed{\int u\,dv = (u \cdot v) - \int v\,du}$ IMP

$\Rightarrow \int u\,dv = uv - \int v\,du$

Combination of one fund. & derivative of another fund.

$\int xe^x dx$ let $u = x \Rightarrow du = dx$
$\int dv = \int e^x dx \Rightarrow v = e^x$

$\Rightarrow \int xe^x dx = xe^x - \int e^x dx \Rightarrow xe^x - e^x + C$

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L = Log
I = Inverse funct.
P = Polynomial
E = Exponential
T = Trig.

* $\int x \ln x \, dx$ $\quad u = \ln x \quad dv = x\,dx$

$du = \frac{1}{x} dx \quad V = \frac{x^2}{2}$

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$$

$$\Rightarrow \frac{x^2}{2} \ln x - \frac{1}{2} \frac{x^2}{2} + C$$

$$\Rightarrow \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

* $\int \ln x \, dx$ $\quad u = \ln x \quad dv = dx$

$du = \frac{1}{x} dx \quad V = x$

$$\int \ln x \, dx = x \ln x - \int 1 \, dx$$

$$= \boxed{x \ln x - x + C} \Rightarrow \text{IMP}$$

$\int \tan^{-1}x\,dx \qquad u = \tan^{-1}x \quad dv = dx$

$du = \frac{1}{1+x^2}dx \quad v = x$

$\int \tan^{-1}x\,dx = x\tan^{-1}x - \int \frac{x}{1+x^2}dx$

$= x\tan^{-1}x - \frac{1}{2}\int \frac{1}{u}du \qquad u = 1+x^2, \quad du = 2x\,dx$

$\Rightarrow x\tan^{-1}x - \frac{1}{2}\ln u + C$

$\Rightarrow x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C$

DO OTHERS OURSELVES.

* $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$

* $\int_0^1 \tan^{-1}x\,dx \qquad u = \tan^{-1}x \qquad \int dv = \int dx$

$du = \frac{1}{1+x^2}dx \qquad v = x$

$x\tan^{-1}x\Big|_0^1 - \int_0^1 \frac{x}{1+x^2}dx \qquad u = 1+x^2, \quad du = 2x\,dx$

$\Rightarrow x\tan^{-1}x\Big|_0^1 - \frac{1}{2}\int \frac{1}{u}du$

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X 8597-34199 drutx!ifh
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$(\tan^{-1} 1 - 0) - \frac{1}{2}(\ln 2 - \ln 1)$

$\tan^{-1} 1 \therefore \frac{\pi}{4} - \frac{1}{2}\ln 2$

Chap 9 - Sec 9.2 ⇒ 1, 2, 3, 4, 5, 6, 7, 29, 30,

Pg 126

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

$$\boxed{(f^{-1})'x = \frac{1}{f'[f^{-1}(x)]}}$$

V.V.V. IMP

$$(f^{-1})'x = \frac{1}{f'(f^{-1}(x))}$$

$\int x^2 \cos x \, dx$ $\qquad u = x^2 \qquad dv = \cos x \, dx$

$\qquad du = 2x\,dx \qquad v = \sin x$

$\int x^2 \cos x\,dx = x^2 \sin x - \int 2x \sin x\,dx$

$\qquad = -2\int x \sin x\,dx \qquad u = x \qquad dv = \sin x\,dx$

$\qquad du = dx \qquad v = -\cos x$

$\Rightarrow x^2 \sin x - 2\left[-x\cos x + \int \cos x\,dx\right]$

$\Rightarrow x^2 \sin x + 2x\cos x - 2\int \cos x\,dx$

$\Rightarrow \underline{x^2 \sin x + 2x\cos x - 2\sin x + C}$

10) $\int \cos^{-1} 2x\,dx \qquad u = \cos^{-1} 2x \qquad dv = dx$

$\qquad du = \frac{1 \cdot 2}{\sqrt{1-4x^2}}dx \qquad v = x$

$\int \cos^{-1} 2x\,dx = x\cos^{-1} 2x - \int \frac{-2x}{\sqrt{1-4x^2}}$

$= x\cos^{-1} 2x - \frac{1}{4}\int \frac{du}{\sqrt{u}} \qquad u = 1-4x^2$

$\qquad du = -8x\,dx$

$= x\cos^{-1} 2x - \frac{1}{4}\int (u)^{-1/2}\,du$

$= x\cos^{-1} 2x - \frac{1}{4}\frac{2u^{1/2}}{1} + C$

$= \underline{x\cos^{-1} 2x - \frac{1}{2}\sqrt{1-4x^2} + C}$

X 8597-32056 research!wow
X 8597-

$\int \ln x\,dx = x\ln x - x + C$

* $\int \ln(2x+3)\,dx$ $u = 2x+3$, $du = 2dx$

$\frac{1}{2}\int \ln u\,du$ → The formula.

$\frac{1}{2}[u\ln u - u] + C$

$\Rightarrow \frac{1}{2}[$

* $\int \ln(2x+3)\,dx$ $u = \ln(2x+3)$ $dv = dx$

$du = \frac{2}{2x+3}dx$ $v = x$

$\int \ln(2x+3)\,dx = x\ln(2x+3) - \int \frac{2x}{2x+3}dx$

$\Rightarrow x\ln(2x+3) - \int 1 - \frac{3}{2x+3}\,dx$

$\Rightarrow x\ln(2x+3) - \int 1\,dx + \int \frac{3}{2x+3}dx$

$\Rightarrow x\ln(2x+3) - x + \frac{3}{2}\int \frac{du}{u}$ $u = 2x+3$, $du = 2dx$

$\Rightarrow x\ln(2x+3) - x + \frac{3}{2}\ln|u| + C$

Do prob. 10, 17, 15, 16, 14, 11, 12, 13, 31, 32, 33, 34, 36, 43, 44

9.2 2) $\frac{1}{3}x e^{3x} - \frac{1}{9}e^{3x} + C$

4) $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$ 6) $x \sin^{-1}x + \sqrt{1-x^2} + C$, 30) $(3e^4+1)/4$,

10) $-x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x} - 6e^{-x} + C$

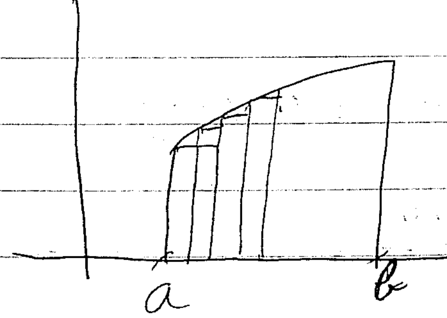
11) $\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$ 32) $(3\sqrt{e}-4)/2e$, 34) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

38) $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

Exp. e^{2x}

L I P E T → Trig

Log, Inv., Polynom. x^3

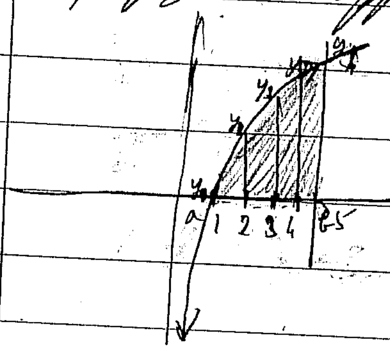


$$\Delta x = \frac{b-a}{N}$$

TRAPEZOIDAL RULE

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2N}\left[y_0 + 2y_1 + 2y_2 + \dots + 2y_{N-1} + y_N\right]$$

Eg → R is the region enclosed by the curve $y = \ln x$, the x-axis & line $x = 5$. If 4 subdivisions of the entire interval $[1,5]$ are used, then the trapezoidal approx. of the area of R is —



$$\Delta x = \frac{5-1}{4} = \frac{4}{4} = 1$$

$$\approx \frac{5-1}{2\cdot 4}\left[0 + 2\ln 2 + 2\ln 3 + 2\ln 4 + \ln 5\right]$$

$$= \frac{1}{2}\left[\ln 4 + \ln 9 + \ln 16 + \ln 5\right]$$

$$= \frac{1}{2}\left[\ln 4\cdot 9\cdot 16\cdot 5\right]$$

$$\approx \underline{3.98}$$

Pg 557 Pg 563, 1, 2

* $f^{-1}(x)' = \dfrac{1}{f'[f^{-1}(x)]}$

$y = f(x) = x^3 + x$

$x = y^3 + y$ $\qquad h'(2)$

~~$2 = y^3 + y$~~

$\therefore \dfrac{dy}{dx} = \dfrac{1}{3y^2+1} \rightarrow$ can't put 2 as it '2' is the $\underline{\underline{x}}$

$\therefore \quad 2 = y^3 + y \Rightarrow \therefore y = 1$

$\Rightarrow \dfrac{1}{3 \cdot 1^2 + 1} = \underline{\underline{\dfrac{1}{4}}}$

* $y = 3 \sin^{-1}(x/2)$

$\dfrac{y}{3} = \sin^{-1}(x/2)$

$\sin \dfrac{y}{3} = \dfrac{x}{2}$

$x = 2 \sin \dfrac{y}{3}$ $\quad$ Inv $\Rightarrow$ $y = 2 \sin \dfrac{x}{3}$

Graph the Inv. only upto a certain point.

INTEGRATION BY PARTS.

$\int e^x \cos x\,dx \qquad u = e^x \quad dv = \cos x$
$\qquad du = e^x dx \quad v = \sin x + C$

$= e^x \sin x - \int \sin x\, e^x dx$

$\qquad u = e^x \quad dv = \sin x$
$\qquad du = e^x dx \quad v = -\cos x$

$\Rightarrow e^x \sin x - \left(-e^x \cos x - \int -e^x \cos x\,dx\right)$

$= e^x \sin x + e^x \cos x + \int -e^x \cos x\,dx$

$= e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$

$+\int e^x \cos x\,dx \qquad\qquad + \int e^x \cos x\,dx$

$\dfrac{2\int e^x \cos x\,dx}{2} = \dfrac{e^x \sin x}{2} + \dfrac{e^x \cos x}{2} + \dfrac{C}{2} \Rightarrow$ divide by 2

$\Rightarrow \int e^x \cos x\,dx = \dfrac{e^x \sin x}{2} + \dfrac{e^x \cos x}{2} + C$

$*\int e^x \sin 3x\,dx$ $\quad u=e^x \quad du=e^x dx \quad dv=\sin 3x \quad V=\frac{1}{3}\cdot\cos 3x \quad V=-\frac{1}{3}\cos 3x$

$\Rightarrow \int e^x \sin 3x\,dx = \frac{1}{3}e^x\cos 3x - \int -\frac{e^x}{3}\cos 3x\,dx$

$\Rightarrow \frac{1}{3}e^x\cos 3x + \frac{1}{3}\int e^x\cos 3x\,dx \quad u=e^x \quad du=e^x dx \quad dv= \quad V=\frac{1}{3}$

$\Rightarrow \frac{1}{3}e^x\cos 3x + \frac{1}{3}\left[\frac{e^x}{3}\sin 3x - \int \frac{e^x}{3}\sin 3x\,dx\right]$

$= \frac{1}{3}e^x\cos 3x + \frac{1}{3}\left[\frac{e^x}{3}\sin 3x - \frac{1}{3}\int e^x\sin 3x\,dx\right]$

$\Rightarrow \frac{1}{3}e^x\cos 3x + \frac{1}{9}e^x\sin 3x - \frac{1}{9}\int e^x\sin 3x\,dx$

$\int e^x\sin 3x = \frac{1}{3}e^x\left(\cos 3x + \frac{1}{3}\sin 3x\right) - \frac{1}{9}\int e^x \sin 3x\,dx$

$\int e^x \sin 3x \qquad + \frac{1}{9}\int e^x \sin 3x\,dx$

$2\int e^x\sin 3x = \frac{1}{3}e^x\cos 3x + \frac{1}{9}e^x\sin 3x$

Pg Sec 9.2 ~~#11~~, 12.

INTEGRATING POWERS OF SINE AND COSINE

... section deals w/ the methods for evaluating ... integrals of the form -

$$\int \sin^m x \cos^n x \, dx$$

where m & n are nonnegative integrals.

and $\int \sin^m x \, dx$ ($n=0$) and $\int \cos^n x \, dx$ ($m=0$)

$$\boxed{\begin{aligned} \cos\left(\frac{\pi}{2}-\alpha\right) &= \sin\alpha \\ \sin\left(\frac{\pi}{2}-\alpha\right) &= \cos\alpha \end{aligned}}$$

~~Sin~~

$$\boxed{\begin{aligned} \tan(\alpha+\pi) &= \tan\alpha \\ \tan(\alpha-\pi) &= \tan\alpha \end{aligned}}$$

$$\boxed{\begin{aligned} \sin\alpha\cos\beta &= \frac{1}{2}[\sin(\alpha-\beta)+\sin(\alpha+\beta)] \\ \sin\alpha\sin\beta &= \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)] \\ \cos\alpha\cos\beta &= \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)] \end{aligned}}$$ ✓

$$\boxed{\begin{aligned} \sin^2 x &= \frac{1}{2}(1-\cos 2x) \\ \cos^2 x &= \frac{1}{2}(1+\cos 2x) \end{aligned}}$$ ✓

V.V. IMP

CASE	PROCEDURE	RELEVANT IDENTITIES
N odd	Sub. $u = \sin x$	$\cos^2 x = 1 - \sin^2 x$
M odd	Sub. $u = \cos x$	$\sin^2 x = 1 - \cos^2 x$
M Even, N Even	Use identities to reduce power on sine & cosine.	$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Sines & Cosines can exist in both even and odd powers and in various combinations -
Unless the problem does fit one of the equations of a double angle or half angle equation or $\cos^2 x$ or $\sin$ or one of the product rules etc., use the following steps

* ALWAYS CONVERT THE VARIABLE OR THE ANGLE IN TERMS OF x, θ, t or u NO COEFFICIENTS

* If Sine is Even and Cos is Odd, then -
TAKE OUT ONE COSINE and MAKE IT EQUAL TO du

* $\int \sin^2 x \cos x \, dx$

$\int \sin^2 x \cos x \, dx$ $\qquad du = \cos x \, dx$

$\int u^2 \, du$ $\qquad \therefore u = \sin x$

$\frac{u^3}{3} + C \Rightarrow \frac{\sin^3 x}{3} + C$

* $\int \sin^6 x \cos^3 x \, dx$

$\int \sin^6 x \cos^2 x \cos x \, dx$ $\qquad du = \cos x \, dx$

$\int \sin^6 x (1 - \sin^2 x) \cos x \, dx$ $\qquad u = \sin x$

$\int u^6 (1 - u^2) \, du \quad \frac{u^7}{7} - \frac{u^9}{9} \Rightarrow \frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C$

$\int u^6 - u^8 \, du \Rightarrow \frac{u^7}{7}$

* $\int \sin^4 x \cos^5 x \, dx$

$\int \sin^4 x \cos^4 x \cos x \, dx$

$\Rightarrow \int \sin^4 x (1-\sin^2 x)^2 \cos x \, dx \qquad du = \cos x \, dx$

$\qquad u = \sin x$

$\int u^4 (1-u^2)^2 \, du$

$\int u^4 (1-2u^2-u^4) \, du$

$\Rightarrow \int u^4 - 2u^6 - u^8 \, du$

* If Sine is Odd and Cosine is Even, then
TAKE OUT ONE SINE AND MAKE IT du

* $\int \sin^3 x \cos^2 x \, dx$

$\int \sin^2 x \cos^2 x \sin x \, dx$

$\Rightarrow \int (1-\cos^2 x) \cos^2 x \sin x \, dx \qquad du = -\sin x \, dx$

$\qquad u = +\cos x$

$-\int (1-u^2) u^2 \, du$

$\Rightarrow -\int u^2 - u^4 \, du$

* $\int \sin^{13} x \cos^4 x \, dx$

$\Rightarrow \int \sin^{12} x \cos^4 x \sin x \, dx \qquad du = \sin x \, dx$

$\int (\sin^2 x)^6 \cos^4 x \sin x \, dx \qquad u = \cos x$

$\Rightarrow \int (1-\cos^2 x)^6 \cos^4 x \sin x \, dx$

$\Rightarrow -\int (1-u^2)^6 u^4 \, du$

If Sine is Odd and Cosine is Odd, then
TAKE OUT A COSINE AND MAKE IT EQUAL TO du.

$\int \cos^5 x \sin x \, dx$

$\int \cos^4 x \sin x \cos x \, dx \qquad du = \cos x \, dx$

$\int (\cos^2 x)^2 \sin x \cos x \, dx \qquad u = \sin x$

$\int (1-\sin^2 x)^2 \sin x \cos x \, dx$

$\int (1-u^2)^2 u \, du$

$\int \cos x \sin x \, dx$

$\int \sin x \cos x \, dx \qquad du = \cos x \, dx$

$\int u \, du \qquad u = \sin x$

$\frac{u^2}{2} + C \Rightarrow \frac{\sin^2 x}{2} + C$

If Both Sine is Even and Cosine is Even, then
USE IDENTITIES TO REDUCE THE POWERS ON SIN & COS.

$\int \sin^2 x \cos^2 x \, dx$

$\int \sin^5 x\,dx$

$\int \sin^4 x \sin x\,dx$ $\quad du = -\sin x\,dx$

$u = \cos x$

$\int (\sin^2 x)^2 \sin x\,dx$

$\int (1-\cos^2 x)^2 \sin x\,dx$

$-\int (1-u^2)^2\,du$

$\Rightarrow -\int 1 - 2u^2 + u^4\,du$

If Sine is Odd, then take out $\sin x = du$

$\int \sin^{11} x\,dx$

$\int \sin^{10} x \sin x\,dx$ $\quad du = -\sin x\,dx$

$u = \cos x$

$\int (\sin^2 x)^5 \sin x\,dx$

$\int (1-\cos^2 x)^5 \sin x\,dx$

$\int (1-u^2)^5\,du$

$\int \cos^3 x\,dx$

$\int \cos^2 x \cos x\,dx$ $\quad du = \cos x\,dx$

$u = \sin x$

$\int (1-\sin^2 x) \cos x\,dx$

$\int (1-u^2)\,du$

If Cosine is Odd, then take out $\cos x = du$

$\int \cos^4 x \, dx$

$= \int (\cos^2 x)^2 \, dx$

$= \int \left[\frac{1}{2}(1+\cos 2x)\right]^2 dx$

$= \frac{1}{4}\int (1+2\cos 2x+\cos^2 2x)\, dx$

$= \frac{1}{4}\int \left(\frac{3}{2}+2\cos 2x+\frac{1}{2}\cos 4x\right) dx$

$\Rightarrow \frac{3}{8}x+\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x + C$

If sin is even or if cos is even, then use the identities to reduce the powers of sin & cos.

Integrals in the form of

$\boxed{\int \sin mx \cos nx \, dx}$ or $\boxed{\int \sin mx \sin nx \, dx}$

or

$\boxed{\int \cos mx \cos nx \, dx}$

we use the PRODUCT FORMULAS

INTEGRATING POWERS OF SECANT AND TANGENTS

$\boxed{\int \tan x \, dx}$

$\Rightarrow \int \frac{\sin x}{\cos x} dx \qquad u = \cos x$

$\qquad\qquad\qquad\qquad du = -\sin x \, dx$

$\Rightarrow -\int \frac{du}{u} \Rightarrow -\ln|u| + C$

$\Rightarrow -\ln|\cos x| + c \Rightarrow \ln|\cos^{-1} x| + c \Rightarrow \ln\left|\frac{1}{\cos x}\right| + c$

$\Rightarrow \boxed{\ln|\sec x| + C}$

$\boxed{\int \sec x \, dx}$

$\int \sec x \, \frac{\sec x + \tan x}{\sec x + \tan x} dx$

$\Rightarrow \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx \Rightarrow \qquad u = \sec x + \tan x$

$\qquad\qquad\qquad\qquad du = \sec x \tan x + \sec^2 x$

$\Rightarrow \int \frac{du}{u} \Rightarrow \ln|u| + C$

$\Rightarrow \boxed{\ln|\sec x + \tan x| + C}$

$\int \sec^2 x = \tan x + C$

$\int \sec^4 x \Rightarrow \int \sec^2 x \cdot \sec^2 x \, dx \qquad u = \tan x$

$\qquad\qquad\qquad\qquad du = \sec^2 x$

$\int (1 + \tan^2 x) \sec^2 x \, dx$

$\Rightarrow \int 1 + u^2 \, du \Rightarrow u + \frac{u^3}{3} + C$

$\Rightarrow \tan x + \frac{\tan^3 x}{3} + C$

X 9119-13760	whamg!jym
X 9119-14855	hoqax!sem
X 9119-16332	lzsc!djk
X 9119-20546	mhwpa!ttc
X 9119-20616	gummo!brb
X 9119-21121	hrmso!mth
X 9119-33135	neptune!ptd
X 9119-35915	gauss!jbs
X 9119-35916	mtunh!klw
X 9119-37152	zeppo!gpv
X 9119-38517	mtsol!ewi
X 9119-82503	mrspock!brf
X 9119-91589	homxb!gjt
X 9119-92309	arch3!mtd
X 9119-94448	hogpe!3723nts
X 9119-94750	hogpa!ftk
X 9119-95708	kato!bobz
X 9119-97975	vax135!ees
X 9122-11299	esun!dgl
X 9122-14049	moss!rgp
X 9122-17044	hos1cad!cje
X 9122-21325	hoqax!bak
X 9122-24319	alux2!tsy
X 9122-26158	hrollie!rcp
X 9122-26308	fjtlc!jbk
X 9122-33882	hogpa!schneid
X 9122-34958	mtuxo!mjpb
X 9122-37688	aloft!dgv
X 9122-85258	sparkey!rxc
X 9122-87930	wmsa!donato
X 9122-88220	alice!dbq
X 9122-93466	io!jdh
X 9122-95479	mtqua!dcj
X 9122-98242	mantic!wab
X 9123-80896	physics!tzeng
X 9124-20788	spin!nuss
X 9124-29403	hugo!ralston
X 9124-88772	spin!lalita
X 9125-11474	mtsol!rdpmp
X 9125-14962	alice!jtp
X 9125-17946	hogpa!van
X 9125-24582	hugo!mallon
X 9125-30938	hogpa!sibilia
X 9125-37699	erebus!ckh
X 9126-36582	neural!edi
X 9127-10025	hogpa!alice
X 9127-10731	mtgzfs3!mike
X 9127-12029	hogpa!jfl
X 9127-12828	hogpa!nm2
X 9127-14699	whamr!cb
X 9127-18230	alux2!hfc
X 9127-18528	whamt!bep
X 9127-18748	mtunh!sa
X 9127-21707	vax135!jho
X 9127-22009	hrcms!ssf
X 9127-22023	hogpa!mekawi
X 9127-24537	hogpa!imh
X 9127-25585	houxa!mcd1
X 9127-26874	attibr!attmxc!cj
X 9127-39232	honet5!nrv
X 9127-80340	research!ees
X 9127-82413	hotsb!rgk
X 9127-83044	ios!len
X 9127-83077	uhura!apw
X 9127-85368	allegra!wdr
X 9127-87020	mink!jjc
X 9127-87134	hosv1!casjeg
X 9127-87565	houxa!betta
X 9127-97622	ihlpl!tep
X 9128-13077	mantic!dgcm
X 9128-26260	floyd!smw
X 9128-30000	hound!fyl
X 9128-88280	mvuca!rock

* If sec = Even, pull $\sec^2 x$ away, change everything else to tan x, let $u = \tan x$, $du = \sec^2 x$.

* If sec is odd and tan is odd pull sec aside and make it du, change everything else to sec. Let $u = \sec x$, $du = \sec x \tan$

* sec is odd and tan is even, change everything else to secants and use a reduction formula.

REDUCTION FORMULA

$$\boxed{\int \sec^N x\,dx = \frac{\sec^{N-2} x \tan x}{N-1} + \frac{N-2}{N-1}\int \sec^{N-}}$$

$$\boxed{\int \tan^M x\,dx = \frac{\tan^{M-1} x}{M-1} - \int \tan^{M-2} x\,dx}$$

* $\int \sec x \tan^2 x\,dx$

$\int \sec x(\sec^2 x - 1)\,dx$

$\int \sec^3 x - \sec x\,dx$

$\int \sec^3 x\,dx - \int \sec x\,dx$

$\left[\frac{\sec x \tan x}{2} + \frac{1}{2}\int \sec x\,dx\right] - \int \sec x\,dx$

$\frac{\sec x \cancel{\tan x}}{2} + \frac{1}{2}\ln(\sec x + \tan x) - \ln|\sec x + \tan$

* Can't use the Reduction fo[rmula] unless it is [in] terms of x, e

$\int \sec^3 x \tan^3 x\, dx$

$\int \sec^2 x \tan^2 x \sec x \tan x\, dx$

$\int \sec^2 x(\sec^2 x - 1) \sec x \tan x\, dx$ $\quad du = \sec x \tan x\, dx$

$\int (\sec^4 x - \sec^2 x) \sec x \tan x\, dx$ $\quad u = \sec x$

$\int (u^4 - u^2)\, du$

$\int \tan^5\theta \sec\theta\, d\theta$

$\int \tan^4\theta \tan\theta \sec\theta\, d\theta$ $\quad du = \sec\theta \tan\theta\, d\theta$

$\int (\tan^2\theta)^2 \tan\theta \sec\theta\, d\theta$ $\quad u = \sec\theta$

$\int (\sec^2\theta - 1)^2 \tan\theta \sec\theta\, d\theta$

$\int (u^2 - 1)^2\, du$

$$\boxed{\begin{aligned} \int \cot x\, dx &= \ln|\sin x| + C \\ \int \csc x\, dx &= \ln|\csc x - \cot x| + C \end{aligned}}$$

Imp

Sec is Even	Sec is Odd	Tan is even	Tan is odd
take out $\sec^2 x$ to make it dx; convert everything else to $\tan x$	use the reduction formula.	Convert tangent to secants & solve	Convert the tangent to the odd power to $\tan^2$'s & tangent to one power & then distribute & solve.

X 9108-87782 hotsb!eco
X 9108-89155 whamt!mh
X 9108-89634 whamt!jat
X 9108-89683 hogpa!mdw
X 9108-91493 esun!wlc
X 9108-94790 fabulus1!chen
X 9108-98549 piccolo!rfk
X 9109-14412 mtsol!jjs2
X 9109-16140 homxb!hlb
X 9109-16210 physics!atf
X 9109-21400 homxb!gcr
X 9109-22840 aluxpo!reh
X 9109-24425 hogpa!whray
X 9109-24572 hocpb!npp
X 9109-26020 lcuxlm!bbb
X 9109-27987 homxb!kls
X 9109-28718 luna!sis
X 9109-30650 moss!jvb
X 9109-31059 hoqax!vgs
X 9109-39429 honet6!hsc
X 9109-81761 arch3!ahk1
X 9109-85635 cblpe!rew
X 9109-93972 groucho!cbk
X 9109-97558 hoh-1!gjf
X 9109-97611 hoh-1!hmp
X 9109-97812 att!uslunix!jss
X 9112-27321 mozart!filip
X 9112-37083 mtung!lav
X 9112-38083 terra!rxg
X 9112-38297 houxa!fred
X 9112-80237 somerset!smst1c!mbg
X 9112-80252 hlwpi!mmiddleton
X 9112-81007 hou2h!psd
X 9112-81353 mtsol!pab3
X 9112-81746 kato!pat
X 9112-84990 hogpa!jde
X 9112-86527 homxb!tar1
X 9112-93843 att!cimu03!jpg
X 9112-95775 mtgzfs3!wchjr
X 9112-97725 hoqax!tjb
X 9113-98726 spin!igal
X 9114-24210 research!jmk
X 9114-26595 aloft!krl
X 9114-39145 hoh-1!rav
X 9114-80384 hogpa!rnpaul
X 9114-89746 hoqaa!orpheus
X 9114-94457 hou2h!gryph
X 9114-95886 hogpa!asun
X 9114-97623 gummo!sy
X 9115-17679 hotsb!foa
X 9115-29973 torreys!ejm
X 9115-30206 druhi!ejb
X 9116-16835 hocus!linos
X 9116-20961 hoh-1!mz
X 9116-96065 homxb!sm3
X 9117-16418 pegasus!pao
X 9117-28919 mtunh!april
X 9117-32599 edsel!wcb
X 9117-81618 mtqua!sds
X 9117-82484 hlwpg!vms
X 9117-83373 lcuxlp!rjz
X 9117-98440 mhcnet!schevon
X 9117-98837 hostar!patmo
X 9118-11944 whamt!vera
X 9118-13123 mtgzfs3!giz
X 9118-14006 hlwpg!ima
X 9118-20400 mtsol!lisa
X 9118-31937 honet5!new
X 9118-32693 ckuxb!jab
X 9118-35138 hoqax!ajv
X 9118-98017 edsel!kec
X 9119-10119 mtqua!ben
X 9119-10797 mvuxd!wct

TRIGONOMETRIC SUBSTITUTIONS

1) $\sqrt{x^2+a^2}$ use $x = a\tan\theta$ $\frac{-\pi}{2} < \theta <$

2) $\sqrt{a^2-x^2}$ use $x = a\sin\theta$ $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

3) $\sqrt{x^2-a^2}$ use $x = a\sec\theta$ $0 \le \theta < \frac{\pi}{2}$ or $\pi \le$

USED FOR

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

* DO NOT LEAVE THE ANSWER IN TERMS OF $\sin\theta$, $\cos\theta$ etc.

INTEGRALS INVOLVING ax^2+bx+c

* $$\int \frac{dx}{x^2-2x+5} \Rightarrow \int \frac{dx}{(x^2-2x+1)+5-1}$$

USE COMPLETING THE SQUARE METHOD IMP

$$\Rightarrow \int \frac{dx}{(x-1)^2+4} \qquad u = x-1, \quad du = dx$$

$$\int \frac{du}{u^2+4} \Rightarrow \frac{1}{2}\tan^{-1}\frac{u}{2}+C$$

* TO USE COMPLETING THE SQUARE WE HAVE TO HAVE ONE ~~AS~~ AS THE COEFFICIENT OF x^2. IMP

* $$\int \frac{dx}{\sqrt{5-4x-2x^2}} \Rightarrow \int \frac{dx}{\sqrt{5-2(x^2+2x)}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{5+2-2(x^2+2x+1)}} \Rightarrow \int \frac{dx}{\sqrt{7-2(x+1)^2}}$$

$u = x+1$

$du = dx$

$$\Rightarrow \int \frac{du}{\sqrt{7-2u^2}} \Rightarrow \int \frac{du}{\sqrt{2}\sqrt{\frac{7}{2}-u^2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \int \frac{du}{\sqrt{\frac{7}{2}-u^2}} \Rightarrow \frac{1}{\sqrt{2}} \sin^{-1}\frac{u\sqrt{2}}{\sqrt{7}}$$

X 8112-70743 hugo!alok
X 8112-71622 whamt!yuhan
X 8116-35759 ihspb!ajg
X 8117-75845 karin!setter
X 8120-77790 cbvox!bruce
X 8124-21262 ihspb!tschow
X 8124-25580 cblph!mes
X 8124-49517 gauss!alon
X 8124-54298 vax135!epdp
X 8124-58627 whamr!kho
X 8126-68363 qsun!rio
X 8126-96321 edsel!phn
X 8126-98828 mink!ajm
X 8127-61029 hogpa!raj2
X 8128-02868 mtnet1!ylee
X 8128-69035 buckaroo!shyam
X 8129-43665 torreys!sheilab
X 8130-52489 honet7!pamb
X 8131-26466 hogpa!bdw76
X 8132-17672 drutx!ecj
X 8132-42172 hotld!lco
X 8133-48394 research!mumick
X 8133-99774 ihspa!meera
X 8134-62789 ulysses!kje
X 8134-74588 boole!ender
X 8135-56783 physics!mas
X 8136-35702 cbosgd!eam
X 8137-76278 qsun!dpk1
X 8137-95109 ho1focus!jcn
X 8138-99442 whamt!hans
X 8142-96378 homxb!cheu
X 8143-19727 mtuxo!raineri
X 8143-41340 mhcnet!uc
X 8143-49546 vax135!kj
X 8143-85162 vax135!skg
X 8143-87595 mhcnet!mjyw
X 8144-85188 druks!wolpert
X 8144-88200 cbvox!gch
X 8147-29471 mtgzy!hcj
X 8148-21506 mtnms!slick
X 8148-62244 buckaroo!kin
X 8152-02662 hocus!cab
X 8152-26460 torreys!rrg
X 8152-44098 homxb!achc
X 8152-49203 mtnet1!wjl
X 8152-97825 library!ate
X 8154-84659 aluxpo!kane
X 8155-76042 mrspock!wjk
X 8156-37263 research!bart
X 8156-46369 drutx!alvin
X 8156-71021 mrspock!eddysan
X 8157-23150 iexist!hhall
X 8158-66616 hound!cwinter
X 8158-95492 honet7!hrw
X 8159-42600 hogpa!jmil
X 8164-12259 arch3!jtm1
X 8165-04847 drutx!huh
X 8166-58695 goofy!gerhard
X 8166-92929 homxb!drk
X 8167-79001 mvuts!vma
X 8168-60354 research!trevor
X 8168-63664 aluxpo!plc
X 8171-19864 hoqub!lja
X 8172-38002 blink!grt
X 8172-71430 hoh-1!pauline
X 8172-94952 hogpa!nyg
X 8173-35188 pikespk!miyamoto
X 8173-46889 hogpa!rhy
X 8173-47507 gauss!clive
X 8173-59255 mtnet1!esh
X 8173-97745 mozart!dsa
X 8174-81584 taz!rjs
X 8174-96919 ihlpl!baral

$* \int \frac{x}{x^2-4x+8}\,dx \Rightarrow \int \frac{x}{(x^2-4x+4)+8-4}\,dx$

$\Rightarrow \int \frac{x}{(x-2)^2+4}\,dx$ $\quad u=x-2$, $du=dx$

$\int \frac{u+2}{u^2+4}\,du \Rightarrow \int \frac{u}{u^2+4}\,du + \int \frac{2}{u^2+4}\,du$

$\Rightarrow z=u^2+4$
$dz=2u\,du$

$\frac{1}{2}\int \frac{dz}{z} + 2\int \frac{du}{u^2+4}$

$\Rightarrow \frac{1}{2}\ln|z| + \cancel{2}\times\frac{1}{\cancel{2}}\tan^{-1}\frac{u}{2}$

$\Rightarrow \frac{1}{2}\ln|u^2+4| + \tan^{-1}\frac{u}{2}$

$\Rightarrow \frac{1}{2}\ln|(x-2)^2+4| + \tan^{-1}\frac{x-2}{2}$

$\Rightarrow \frac{1}{2}\ln(x-2)^2+4 + \tan^{-1}\frac{x-2}{2}$

$*\ \int \frac{2x+5}{x^2+2x+5}\,dx$

$\Rightarrow \int \frac{2x+5}{(x^2+2x+1)+5-1}\,dx \Rightarrow \int \frac{2x+5}{(x+1)^2+4}\,dx \qquad u = x+1,\ du = dx$

$\Rightarrow \int \frac{2(u-1)+5}{u^2+4}\,du \Rightarrow \int \frac{2u+3}{u^2+4}\,du \Rightarrow \int \frac{2u}{u^2+4}\,du + \int \frac{3}{u^2+4}\,du$

$z = u^2+4$
$dz = 2u\,du \Rightarrow \int \frac{dz}{z} + 3\int \frac{1}{u^2+4}$

$\Rightarrow \ln|z| + \frac{3}{2}\tan^{-1}\frac{u}{2} + c$

$\Rightarrow \ln(u^2+4) + \frac{3}{2}\tan^{-1}\frac{x+1}{2} + c$

$\Rightarrow \ln[(x+1)^2+4] + \frac{3}{2}\tan^{-1}\frac{x+1}{2} + c$

---

* PROPER RATIONAL FUNCTIONS → degree of the numerator is smaller than the degree of the denominator.

* This method is good for proper rational functions and factorable denominator.

$\int \frac{dx}{x^2+x-2} = \int \frac{dx}{(x+2)(x-1)}$

$\frac{1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)} \Rightarrow \frac{1}{(x+2)(x-1)} = \frac{A(x-1)+B(x+2)}{(x+2)(x-1)}$

$\Rightarrow \frac{1}{(x+2)(x-1)} = \frac{Ax-A+Bx+2B}{(x+2)(x-1)} \Rightarrow \frac{1}{(x+2)(x-1)} = \frac{(A+B)x+2B-A}{(x+2)(x-1)}$

$A+B=0$
$-A+2B=1$
$B = 1/3$

$\therefore A = -1/3$

X 8034-80587 mtqua!drr
X 8035-34646 mtqua!tvl
X 8035-38906 polaris!chong
X 8037-31315 dwrock!vqh
X 8037-35663 aluxpo!ctt
X 8037-84869 luna!tat
X 8037-86876 hogpa!nth
X 8037-93277 att!usl!usl!linh
X 8037-96224 hogpa!nvt
X 8039-31658 ihlpy!qtm
X 8041-20333 mtung!jadoo
X 8060-58393 mvuxd!pt
X 8062-61563 globe2!eag
X 8066-24594 honshu!stv
X 8068-61874 whamg!jmc
X 8068-65125 globe1!lnf
X 8069-82619 hoqub!media
X 8069-82822 corona!cruz
X 8069-89504 mantic!adolfo
X 8079-37282 cbvox!gsl
X 8080-15063 mvuts!jrb
X 8080-22977 troll!rhv
X 8080-37079 lcuxlm!ejh
X 8080-38999 hogpg!wtju
X 8080-41193 iw4os!rickr
X 8080-72731 druwa!rgb
X 8081-21499 druwa!rge
X 8082-25285 cbvox!young
X 8082-33449 cbnea!jec
X 8082-62331 hoqas2!romek
X 8083-14965 holite!klt
X 8083-83991 drutx!rjg
X 8084-84553 goofy!dathomas
X 8085-08605 whamr!jnk
X 8085-20050 physics!prk
X 8085-46334 drutx!rls
X 8085-60325 violin!dmw
X 8085-81804 pixels!bwm
X 8085-92522 drutx!jcb
X 8086-87181 mhuhk!dtch
X 8086-91390 hrollie!susan
X 8087-22934 luna!ec
X 8087-62380 trumpet!wdl
X 8087-83236 mtuxo!cbp
X 8087-94632 druwa!dcg
X 8087-98155 vax135!tll
X 8088-14246 cbnmva!batu
X 8088-87440 hulk!knw
X 8089-05732 mvuts!jwang
X 8089-27769 edsel!trix
X 8090-52066 edsel!tcj
X 8091-29187 iwtin!jrw
X 8094-11669 longs!cme
X 8094-19926 hoqub!hmk
X 8094-96450 mvuas!bu
X 8095-70585 ihlpf!marvin
X 8098-17532 ihlpf!nolen
X 8099-46053 ihlpf!kdh
X 8099-46540 homxb!cym
X 8099-46640 hotlan!sjd
X 8099-47683 ihlpf!wilks
X 8099-64986 hugo!pjs
X 8102-30065 ihlpf!pax
X 8103-23838 hogpa!holm
X 8103-46461 hugo!tzung
X 8104-34587 druwa!bph
X 8104-54289 iw4os!ranjan
X 8106-48716 alice!ephraim
X 8107-77039 arch2!matin
X 8108-10517 hoqaa!rlarkin
X 8108-98779 polaris!nrt
X 8109-60322 cbnea!kcs
X 8111-10309 arch3!gxl

$$\int \frac{dx}{x^2+x-2} = \int \frac{dx}{(x+2)(x-1)}$$

$$\int \frac{-1/3}{x+2}\,dx + \int \frac{\frac{1}{3}}{x-1}\,dx$$

$$\Rightarrow -\frac{1}{3}\ln|x+2| + \frac{1}{3}\ln|x-1| + C$$

$$\Rightarrow -\frac{1}{3}\ln\frac{|x+2|}{|x-1|} + C$$

$$* \int \frac{dx}{(x+2)^2(x-1)} \Rightarrow$$

$$\frac{1}{(x+2)^2(x-1)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$* \int \frac{dx}{x^3(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{(x+1)}$$

$$* \int \frac{2x+4}{x^3-2x^2} = \int \frac{2x+4}{x^2(x-2)}$$

$$\frac{2x+4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-2)}$$

$$\frac{Ax^2-2Ax+Bx-2B+Cx^2}{x^2(x-2)}$$

$$\Rightarrow \frac{x^2(A+C)+x(B-2A)-2B}{x^2(x-2)}$$

$$\frac{0x^2+2x+4}{x^2(x-2)} = \frac{(A+C)x^2+(-2A+B)x-2B}{x^2(x-2)}$$

$$A+C=0$$
$$-2A+B=2$$
$$-2B=4$$
$$B=-2,\ A=-2,\ C=2$$

$$\int -\frac{2}{x}\,dx + \int \frac{-2}{x^2}\,dx + \int \frac{2}{x-2}\,dx$$
$$= -2\ln|x| + \frac{2}{x} + 2\ln|x-2| + C$$

INTEGRATING RATIONAL FUNCTIONS AND PARTIAL FUNCTIONS

* $$\int \frac{5x-4}{x^2-4x}\,dx \Rightarrow \int \frac{5x-4}{x(x-4)}\,dx$$

$$\Rightarrow \frac{5x-4}{x(x-4)} = \frac{A}{x} + \frac{B}{(x-4)}$$

$$\Rightarrow \frac{5x-4}{x(x-4)} = \frac{Ax-4A+Bx}{x(x-4)} = \frac{(A+B)x-4A}{x(x-4)}$$

$$A+B=5$$
$$-4A=-4$$
$$A=1,\ B=4$$

$$\int \frac{5x-4}{x^2-4x}\,dx = \int \frac{1}{x}\,dx + \int \frac{4}{x-4}\,dx$$
$$= \ln|x| + 4\ln|x-4|$$

* $$\int \frac{x^2+x-2}{3x^3-x^2+3x-1}\,dx$$

$$\Rightarrow \int \frac{x^2+x-2}{x^2(3x-1)+(3x-1)}\,dx \Rightarrow \int \frac{x^2+x-2}{(3x-1)(x^2+1)}\,dx$$

IRREDUCIBLE QUADRATIC

X 7982-96915 hoqub!wrw
X 7982-98999 allegra!jok
X 7984-94840 hrmso!lrv
X 7985-27852 homxb!gfe
X 7985-84397 mtqua!lwd
X 7987-14087 research!eowyn
X 7987-28652 lcuxlm!jgm
X 7987-29803 hogpa!msp
X 7987-32196 hopdb!paul
X 7987-39295 mozart!msg
X 7987-93576 hogpa!akadeke
X 7987-98107 pruxp!rbl
X 7988-19331 mink!scm
X 7988-30556 hotlan!yhd
X 7988-30992 hogpa!jbutz
X 7988-32865 hotsoup!agk
X 7988-88179 cbnmva!mjw
X 7988-90732 homxb!eileen
X 7989-30914 whamt!ffk
X 7989-37118 hoh-1!tsc
X 7989-37563 mtnet1!sle
X 7992-16621 lcuxlm!harry
X 7992-18501 fjtld!dl1
X 7992-80169 mtdcr!kgm
X 7992-96020 hrmso!peter
X 7992-98638 hocpb!jrrt
X 7995-24171 mtqua!hmz
X 7995-38175 mtnet1!rob
X 7997-26452 druwa!rcb
X 7997-27495 abars!sab
X 7997-28847 physics!spstrong
X 7997-35788 mvuxd!rxb
X 7997-95297 aloft!bng
X 7997-97398 petrel!jgehlin
X 7998-15220 houxa!mam1
X 7998-37275 hogpa!umesh
X 7998-38430 att!sfsup!hmj
X 7998-39075 anuxt!nkb
X 7998-83131 arch3!manuel
X 7998-90646 violin!dac
X 7999-14158 hotld!wew
X 7999-19863 hogpa!wfa
X 7999-20126 gummo!plb
X 7999-24788 ohm!sd
X 7999-24828 hoqaa!pbg
X 7999-25507 violin!rth
X 8000-06502 drutx!max
X 8004-49823 esun!jose
X 8004-97595 torreys!izagma
X 8005-10984 alux2!jefig
X 8006-73131 hogpa!carolc
X 8007-53825 globe1!jln
X 8009-61549 hogpa!whlec
X 8009-80057 globe1!jose12
X 8010-54625 whamt!car
X 8012-61687 mvuts!cecil3
X 8020-50257 honet7!javier
X 8020-64454 hogpa!radar
X 8025-60103 mtsol!jaime
X 8028-57193 whamt!maria
X 8030-18670 hotsc!lmd
X 8030-31737 hotlg!anh
X 8030-33092 mink!ttt
X 8030-84807 esun!jpeng
X 8030-86384 hotlg!ntn
X 8032-20363 ulysses!dbn
X 8032-21868 cblph!tvd
X 8032-25279 aloft!hqp
X 8032-30288 woomera!ttl
X 8032-30971 mvucl!kim
X 8032-36904 yen!avm
X 8032-84426 hotlg!lsd
X 8032-96193 mvuye!lcn

$$\frac{n^2+n-2}{(3n-1)(n^2+1)} = \frac{A}{3n-1} + \boxed{\frac{Bn+C}{n^2+1}}$$

In case of an Irreducible Quadratic we use the format $Bn+C$

$$\Rightarrow \frac{A(n^2+1)+(Bn+C)(3n-1)}{(3n-1)(n^2+1)}$$

$$\Rightarrow \frac{An^2+A+3Bn^2-Bn+3Cn-C}{(3n-1)(n^2+1)}$$

$$\Rightarrow \frac{n^2(A+3B)+n(3C-B)+A-C}{(3n-1)(n^2+1)}$$

$\therefore$ $3C-B=1$ $\quad A=C-2$

$A+3B=1$

$A-C=-2$ $\quad C-2+3B=1$

$C+3B=3$

$3C-B=1$ $\quad 3C-B=1$

$3C+1=1$ $\quad -3C-9B=-9$

$C=0$ $\quad 3C-B=1$

$\therefore A=0-2=-2$ $\quad -10B=-8$

$B=\frac{8}{10}$

$$\int \frac{-7/3}{3n-1}\,dn + \int \frac{4/5\,n+3/5\,dn}{n^2+1}$$

PROPER FUNCTION → the denominator has to be larger than the degree of the numerator.

$$\int \frac{3x^4+3x^3-5x^2+x+1}{x^2+x-2} \rightarrow \text{Improper function}$$

$$\begin{array}{r|l} & 3x^2+1 \\ \hline x^2+x-2 & 3x^4+3x^3-5x^2+x+1 \\ & \underline{3x^4+3x^3-6x^2} \\ & x^2+x+1 \\ & \underline{x^2+x-2} \\ & 3 \end{array}$$

$$\Rightarrow \int 3x^2+1\,dx + \int \frac{3}{x^2+x-2}\,dx$$

$$\Rightarrow \int 3x^2+1\,dx + \left[\int \frac{3}{x^2+x-2} = \frac{A(x-1)+(B(x+2))}{(x+2)(x-1)}\right]$$

$$\Rightarrow \frac{Ax+Bx-A+2B}{(x+2)(x-1)} = \frac{(A+B)x+2B-A}{(x+2)(x-1)}$$

$$A+B=0$$
$$-A+2B=3$$
$$3B=3$$
$$B=1 \quad \therefore A=-1$$

$$\Rightarrow \int 3x^2+1\,dx + \int \frac{-1}{x+2}\,dx + \int \frac{1}{x-1}\,dx$$

$$= x^3+x-\ln\frac{|x-1|}{|x+2|}+c$$

X 7949-84969 honet4!jsr1
X 7949-87823 gummo!larry
X 7952-15368 research!wcohen
X 7952-30017 cbcat!sbt
X 7952-31280 whamt!rld
X 7952-31503 hogpa!trev
X 7952-38917 angate!cohen
X 7952-97941 mtnet1!pnh
X 7954-17601 alice!evelyne
X 7954-90669 karin!fschu
X 7954-97700 whamr!jia
X 7954-98165 arch3!raj
X 7955-14694 hocpa!hk
X 7955-35555 honet4!rjc3
X 7957-18345 whamt!mao
X 7957-83517 mtdcr!nhc
X 7957-83624 terra!ser
X 7958-10206 hogpa!cho
X 7958-82303 lzusp!pwg
X 7958-85304 ulysses!boris
X 7958-95981 mink!ashwin
X 7958-97162 homxb!lida
X 7958-98887 physics!mihai
X 7959-18949 whamg!rff
X 7959-20771 hostar!mitch
X 7959-21122 hogpa!stekas
X 7959-23229 mtnet1!rosey
X 7959-94790 hogpa!hojjr
X 7962-32206 mtgzfs3!jpa
X 7962-38558 hostc2!kenz
X 7962-38906 mtuxo!pmb
X 7962-93445 mink!csy
X 7962-99729 hotsoup!dld
X 7965-10925 mtfmi!habib
X 7967-17673 lcuxlm!esc
X 7967-20412 hogpa!jims
X 7967-26646 anchor!wwn
X 7967-28140 mtnet1!lic
X 7967-29543 hrmso!ph
X 7967-89875 hogpa!kevint
X 7967-95572 lcuxlq!dale
X 7968-10548 homxb!jbs1
X 7968-14668 homxb!step
X 7968-80304 hotseat!okw
X 7968-89261 mtdcr!vfried
X 7969-18097 probe!esmall
X 7969-24905 fjtle!gfh
X 7969-25535 aluxpo!wlb
X 7969-26399 fjtlc!lennyg
X 7969-32415 mvuxd!car
X 7969-32458 hogpa!whjcs
X 7969-34080 hrmso!rwj
X 7969-34348 hound!jxc
X 7969-38391 picard!rtp
X 7969-84578 houxa!frankb
X 7969-84727 alice!dab
X 7972-14705 polaris!usk
X 7972-23441 hotsb!mjy
X 7972-27689 hoqax!chris
X 7972-97914 arch3!augusto
X 7977-13091 hogpa!bjh
X 7977-36976 buckaroo!arthur
X 7977-93589 mtnet1!jbburke
X 7977-95824 homxb!mc
X 7977-97216 max!scpj
X 7978-28991 mink!jean
X 7978-80267 whamr!omar
X 7978-85117 mozart!bernie
X 7979-14305 mvuxd!a1
X 7979-24169 mtgzy!mkatz
X 7979-27724 houxa!jce
X 7982-13101 hogpa!ebp
X 7982-96500 honet7!rachel

* $\int \frac{dt}{\sqrt{3-4t-4t^2}} \Rightarrow \int \frac{dt}{\sqrt{(3+1)-4(t^2+t+\frac{1}{4})}}$

$\Rightarrow \int \frac{dt}{\sqrt{4-4(t+\frac{1}{2})^2}} \Rightarrow \int \frac{dt}{\sqrt{4(1-(t+\frac{1}{2})^2}}$

$\frac{1}{2} \int \frac{dt}{\sqrt{1-(t+\frac{1}{2})^2}}$ $u = t+\frac{1}{2}$, $du = dt$

$\Rightarrow \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}} \Rightarrow \frac{1}{2} \sin^{-1} t + \frac{1}{2} + C$

* $\int \frac{3n^2+12n+2}{(n^2+4)^2} \Rightarrow$ IS PROPER

V.V.V. IMP PROB.

$\Rightarrow \frac{An+B}{(n^2+4)^2} + \frac{Cn+D}{(n^2+4)}$

$\frac{An+B+(Cn+D)(n^2+4)}{(n^2+4)^2}$

$\Rightarrow \frac{An+B+Cn^3+4Cn+Dn^2+4D}{(n^2+4)^2}$

$\Rightarrow \frac{Cn^3+Dn^2+An+4Cn+B+4D}{(n^2+4)^2}$

$\Rightarrow C=0, D=3$

$A+4C=12, B+4D=2$

$\therefore A=12$ & $\cancel{D}$ $B=-10$

$$\Rightarrow \int \frac{12x-10}{(x^2+4)^2}\,dx + \int \frac{3}{x^2+4}\,dx$$

$$\Rightarrow \int \frac{12x}{(x^2+4)^2}\,dx - \int \frac{10}{(x^2+4)^2}\,dx + 3\int \frac{dx}{x^2+4}$$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\,d\theta$

$$\Rightarrow \frac{-6}{x^2+4} \quad - 10\int \frac{2\sec^2\theta\,d\theta}{(4\tan^2\theta+4)^2} \quad + \frac{3}{2}\tan^{-1}\frac{x}{2}$$

$$\Rightarrow \frac{-20}{16}\int \frac{\sec^2\theta\,d\theta}{\sec^4\theta}$$

$$\Rightarrow -\frac{5}{4}\int \frac{d\theta}{\sec^2\theta}$$

$$\Rightarrow -\frac{5}{4}\int \cos^2\theta\,d\theta$$

$$\Rightarrow -\frac{5}{4}\times\frac{1}{2}\int 1+\cos 2\theta\,d\theta$$

$$\Rightarrow -\frac{5}{8}\left[\theta + \frac{1}{2}\sin 2\theta\right]$$

$$\Rightarrow -\frac{5}{8}\left[\theta + \frac{1}{\cancel{2}}\times\cancel{2}\sin\theta\cos\theta\right]$$

$$\Rightarrow -\frac{5}{8}\left[\tan^{-1}\frac{x}{2} + \frac{x}{\sqrt{x^2+4}} \times \frac{2}{\sqrt{x^2+4}}\right]$$

$$\Rightarrow \frac{-6}{x^2+4} - \frac{5}{8}\tan^{-1}\frac{x}{2} - \frac{5\cancel{10}x}{\cancel{8}4(x^2+4)} + \frac{3}{2}\tan^{-1}\frac{x}{2}$$

$$\Rightarrow \frac{-6}{x^2+4} - \frac{5}{8}\tan^{-1}\frac{x}{2} - \frac{5x}{4(x^2+4)} + \frac{3}{2}\tan^{-1}\frac{x}{2}$$

* ALWAYS TRY TO SOLVE BY PARTIAL DECOMPOSITION RATHER THAN USING THE COMPLETING THE SQUARE METHOD.