Utsav Bali
Calculus Review
1991- 1993

2. FUNCTIONS & LIMITS

[illegible] → A function is a rule that assigns to each element in set A one & only one element in set B.

For a function there is only one value of y for that funct. given a value of x. (Use the vertical line test) → IMP

Domain of a function is the set of all possible x values & } IMP
Range is the set of all possible y values for a function.

Calculus always deals with radian measures.

If a function is defined by a formula & no domain is specified, then it is understood that the domain consists of all real no's.

Upon algebraic simplification, the new functions domain may be altered & so must always justify the domain of the new function.

Piecewise defined functions.

Constant functions → $f(x) = k$ (k is constant)

Polynomial functions → linear, quadratic, cubic etc (always continuous) IMP } IM

RATIONAL FUNCTIONS → functions that can be expressed as ratio of two polynomials. (always continuous except when denominator = 0)

Explicit Algebraic functions → functions that can be evaluated using finitely many addition, substraction, multiplication, division & root extractions.

Log & Trig are transcedental functions. → IMP

$\frac{5\pi}{2}$ ... ⇒ $\frac{\pi}{2} + 2n\pi$ where $n = 1, 2, 3, 4, 5 \ldots$

If f is function of x, then we define the graph of f to be the graph of f
eq $y = f(x)$.

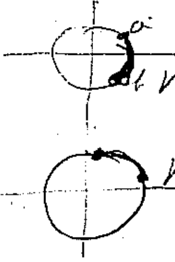
Hole in the graph of a function means the $f(c) = \text{undef.}$ any point } IMP

In a graph don't forget to label x & y intercepts if any.

Verticle line test, tests a graph if it is a function or not. IM

Equations of semi-circle could be given in the form of
[illegible] etc the + & − determine the side which it is going to open

- y explicitly as a function of x → y = … →
- y implicitly as a function of x → x = y → } IMP



not a

not a

- $f(x) = |x| = f(x) = \begin{cases} x, & x > 0 \\ -x, & x < 0 \\ 0, & x = 0 \end{cases}$ } IMP

Horizontal line test → Invertibility → IMP

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(f \times g)(x) = f(x) \times g(x)$
- $(f/g)(x) = f(x)/g(x) \Rightarrow g(x) \neq 0$
- $(f \circ g)(x) = f(g(x))$
- Express a function as a composition of two functions
- odd function is symmetric to the origin & even function is symmetric to the y-axis. $f(-x) = f(x) \rightarrow$ even $f(-x) = -f(x) = $ odd

LIMITS (Intuitive approach)

- differential & Integral Calculus.
- $\lim_{x \to 0} f(x) = L$ if and only if $\lim_{x \to 0^+} f(x) = L = \lim_{x \to 0^-} f(x) = L$ } IMP
- One sided & two sided limits.
- I The limit of a function at a certain point is not related to the value of the function at that point. → IMP
- The left handed & the right handed limits can be different } IMP
- M The left & right handed limits can have values where the function itself is undefined.
- It is possible that the limit of a funct. at a point = the value of the funct. at that point.
- P Limits for oscillating graphs do not exist → IMP eg $\boxed{\lim_{x \to 0} \sin \frac{1}{x}}$
- Continuous & discontinuous functions. * Polynomial functions are always continuous. → IMP

om: mtdcr!rrs
te: Tue Aug 11 09:46 EDT 1992
bject: Dale Harman's Luncheon
: mtdcr!org-45351 /all=yes
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me celebrate Dale Harman's 10th year anniversary!!!
: 8/20/92
ere: Amravathi Indian Restaurant (Buffet Lunch)
me: 12:00 noon
st: $10.00 (includes gift)

lease contact Ray Staab (76179) or Julie Renzi (73411) if interested.

Limit computation

limit of a constant function at any point is the constant no.

$\lim_{x\to \boxed{a}} k = k$ & $\lim_{x\to a^+} k = k$ & $\lim_{x\to a^-} k = k$ → IMP

$\lim_{x\to \boxed{a}} x = a$ → IMP

* $\lim_{x\to \boxed{0^+}} \frac{1}{x}$ & $\lim_{x\to \boxed{0^-}} \frac{1}{x}$ & $\lim_{x\to \boxed{0}} \frac{1}{x} = DNE$ → IMP

* DNE could sometimes be $+\infty$ or $-\infty$.

a) $\lim [f(x) + g(x)] = \lim f(x) + \lim g(x) = L_1 + L_2$

b) $\lim [f(x) - g(x)] = \lim f(x) - \lim g(x) = L_1 - L_2$

c) $\lim [f(x) \times g(x)] = \lim f(x) \times \lim g(x) = L_1 \times L_2$

d) $\lim [f(x) / g(x)] = \lim f(x) / \lim g(x) = L_1/L_2$ $g(x) = L_2 \neq 0$

IMP e) $\lim \sqrt[n]{f(x)} = \sqrt[n]{\lim f(x)} = \sqrt[n]{L_1}$ if $L_1 > 0$ if n is ev

IMP

$\lim (\sin x) = \sin (\lim x)$

$\lim |x| = |$

A constant factor can be moved through a limit sign. → IMP

$\lim kf(x) = \lim k \lim f(x) = k \lim f(x)$

For a polynomial $P(x)$, the limit as ~~x~~ x approaches a is equal the value of the polynomial at a.

To find the limit of a rational function, if the numerator & deno $= \boxed{\frac{0}{0} \text{ or } \frac{\infty}{\infty}}$ then the function is indeterminate if when $x \to x$

The limit of an indeterminate function can be found by either simplifying it or by using the one sided limits if $x \to \infty$.

↓

Synthetic division } IMP

* If the denominator of a rational funct- is $\boxed{0}$ & not the numerator then sol as $\boxed{x \to no}$, then solve the function using one sided limits -> $\boxed{\text{IMP}}$

* If $\lim\limits_{x \to a^+} f(x) \neq \lim\limits_{x \to a^-} f(x)$, then $\lim\limits_{x \to a} f(x) = \text{DNE}$ -> $\boxed{\text{IMP}}$

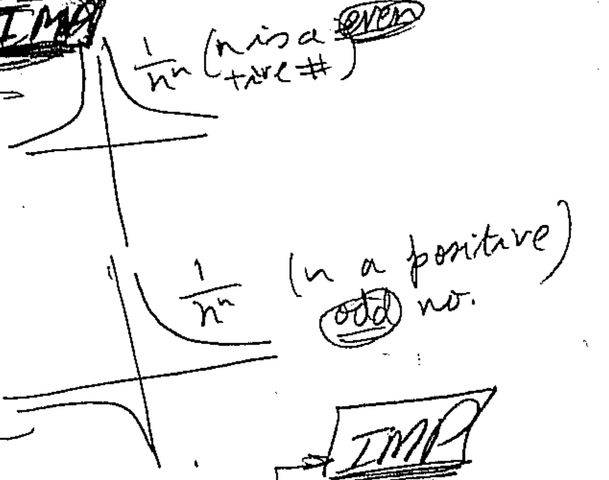
* $\lim\limits_{x \to \boxed{\infty}} \frac{3x^2}{2x^2} = \frac{3}{2}$ $\lim\limits_{x \to \boxed{\infty}} \frac{x}{x^2} = 0$ $\lim\limits_{x \to \boxed{\infty}} \frac{x^2}{x} = \text{DNE}$ -> $\boxed{\text{V.V.IMP}}$

$x \to \infty$

* $\lim\limits_{x \to +\infty} \frac{1}{x^n} = \left(\lim\limits_{x \to +\infty} \frac{1}{x}\right)^n = 0$

* $\lim\limits_{x \to -\infty} \frac{1}{x^n} = \left(\lim\limits_{x \to -\infty} \frac{1}{x}\right)^n = 0$

$\boxed{\text{V.V.V IMP}}$



* whenever $\boxed{x \to \infty}$ be careful of the SIGN ON INFINI

* In a rational function -> if the power in the denominator is greater than the power in the numerator, then the limit = 0.
* If power in numerator is greater than power in denominator then lim = DNE.
* If the power is the same in numerator & denominator, then the limit = the coefficients.

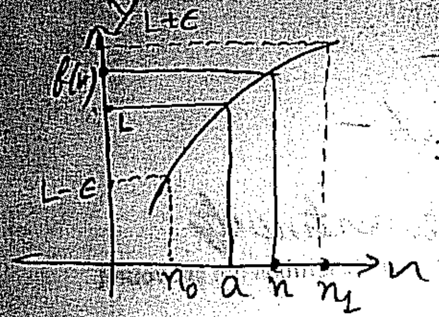
* $\boxed{\sqrt{x^2} = |x|}$, $x, x>0 \to +\infty$; $-x, x<0 \to -\infty$; $0, x=0$ $\boxed{\text{IMP}}$

* Use one side limits to solve piece wise defined functions -> $\boxed{\text{IMP}}$

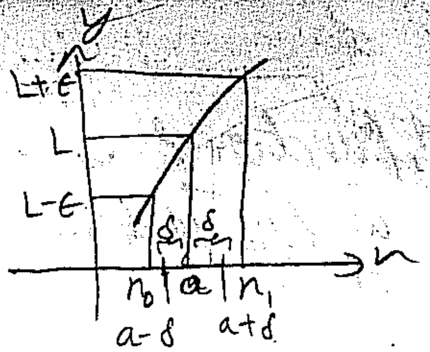
Rigorous approach

* $\lim\limits_{x \to a} f(x) = L$ * If given any number $\epsilon > 0$, we can find a number $\delta > 0$ such that $f(x)$ satisfies $|f(x) - L| < \epsilon$ whenever x satisfied $|x - a| < \delta$

VERY IMP



$= |f(n) - L| < \epsilon$



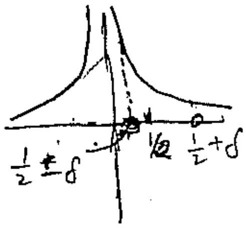
$|n - a| < \delta$

$\lim_{n \to 1/2} \frac{1}{n} = 2$

$\left|\frac{1}{n} - 2\right| < \epsilon \quad ; \quad \left|n - \frac{1}{2}\right| < \delta$

$\left|\frac{2}{n}\left(\frac{1}{2} - n\right)\right| < \epsilon$

$= \left|\frac{2}{n}\right| \left|\left(\frac{1}{2} - n\right)\right| < \epsilon$ now $\left|\frac{2}{n}\right| \to$



$= \left|\frac{2}{n}\right| \left|n - \frac{1}{2}\right| < \epsilon$

assume $\delta = 1/4$

$\therefore \left|n - \frac{1}{2}\right| < \frac{1}{4}$

$= -\frac{1}{4} < n - \frac{1}{2} < \frac{1}{4}$

$= \frac{1}{4} < n < \frac{3}{4}$

$\frac{4}{1} > \frac{1}{n} > \frac{4}{3}$

$8 > \frac{2}{n} > \frac{8}{3}$

$\frac{8}{3} < \frac{2}{n} < 8$

$\therefore -8 < \frac{2}{n} < 8$

$\to \left|\frac{2}{n}\right| < 8$

$\to \quad 8\left|n - \frac{1}{2}\right| < \epsilon$

$\therefore \delta = \frac{\epsilon}{8}$

$\therefore \delta = \min\left(\frac{1}{4}, \frac{\epsilon}{8}\right)$

CONTINUITY

* A function f is continuous at point c if -

a) f(c) is defined
b) $\lim_{x \to c} f(x)$ exist
c) $\lim_{x \to c} f(x) = f(c)$

V.V.V* IMP Points of discontinuity

* This function is continuous at (a,b) → IMP (non-inclusive)

* Polynomials are continuous functions → IMP

$f(x) = x^2 - 2x + 1 \Rightarrow$ ~~c^2~~ $c^2 - 2c + 1 = f(c) = c^2 - 2c + 1$

* $f(x) = |x|$ is continuous as

$$f(x) = \begin{cases} x, & x > 0 \\ -x, & x < 0 \\ 0 = x = 0 \end{cases}$$

~~as lim f(x)~~ $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$ → IMP

* If the functions f & g are continuous then at c then

IMP
- f+g is contd. at c
- f-g is contd at c
- f.g is contd at c
- f/g is contd. at c if $g(c) \neq 0$ & is discontd. at c if g(c

* A rational function is continuous everywhere except at the points where the denominator is zero. → IMP

* $\lim |g(x)| = |\lim g(x)|$ → IMP

MOST IMP: If $\lim g(x) = L$ & if function f is continuous at L then $\lim f(g(x)) = f(L)$ or $\lim f(g(x)) = f(\lim g(x))$

* A function is continuous from the left at the point c if

1) f(c) is defined
2) ~~f~~ $\lim_{x \to c} f(x)$ exists
3) $\lim_{x \to c} f(x) = f(c)$

IMP

Following are additional questions and resposes for risk analysis:

10. Does incomplete RFI adversely impact the outcome of the Inbound Platform Delivery? Yes_X__ NO____

Comments:

RFI for Operations and Billing is not complete.

11. Is there a risk associated with "resistance to change" by process owners that would impact the delivery of the platform?

Yes__X_ NO____

Comments:

A function f is continuous from the right at c if —

1) $f(c)$ exists
2) $\lim\limits_{x \to c^+} f(x)$ exists
3) $\lim\limits_{x \to c^+} f(x) = f(c)$

IMP

IMP = look at ☆ & pg 125 of book

INCLUSIVE

A function is said to be continuous at [a,b] if

1) is contd at (a,b)
2) is continuous at a from the right
3) is continuous at b from the left

IMP

☆ If f is continuous on a closed interval [a,b] & C is a number between $f(a)$ & $f(b)$, inclusive, then there is at le one number x in the interval [a,b] such that $f(x) = c$

INTERMEDIATE VALUE THEOREM → IMP

* If f is continuous ~~at~~ on [a,b] & if $f(a)$ & $f(b)$ have opposite signs, then there is at least one solution of the equation $f(x)=0$ in the interval (a,b).

LIMITS & CONTINUITY OF TRIG. FUNCTIONS

- Area of a sector $= \frac{1}{2} \times r^2 \times \theta$ or $\frac{1}{2} \times r \times s$ → IMP
- Length of an arc $= s = r \times \theta$
- $\lim [\sin(g(n))] = \sin(\lim g(n))$
- $\lim [\cos(g(n))] = \cos(\lim g(n))$

MOST IMP

$\lim |g(n)| = |\lim g(n)|$

- PINCHING AND SQUEEZING THEOREM → IMP

$= g(n) \leq f(n) \leq h(n)$, then if:

IMP

$$\lim_{n \to a} g(n) = \lim_{n \to a} h(n) = L$$

then f also has this limit as n approaches a, i.

$$\lim_{n \to a} f(n) = L \text{ also}$$

INDETERMINATE so USE ONE S

LIM

$$\boxed{\lim_{n \to 0} \sin h = 0}$$

* $$\boxed{\lim_{n \to 0} \cos h = 1}$$

* $$\boxed{\lim_{n \to 0} \frac{\sin h}{h} = 1}$$

* $$\boxed{\lim_{n \to 0} \frac{1-\cos h}{h} = 0}$$

MOST IMP

$$\frac{1-1}{1} = \frac{0}{1} = 0$$

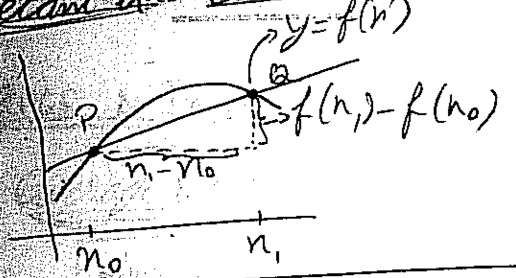
1st derivative & 2nd derivat test.

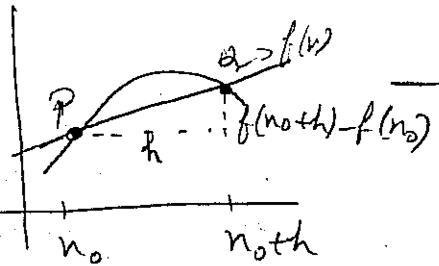
DIFFERENTIATION

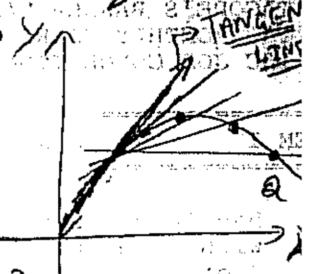
Tangent lines and Rates of Change

If a secant line is drawn between 2 points P & Q on a curve, & Q is allowed ~~to~~ to move along the curve towards P then the secan line rotates towards a limiting position which can be regarded as the tangent line to the curve at that point P.

A secant line cuts a curve at 2 place







$$\boxed{m_{sec} = \frac{f(x_1)-f(x_0)}{x_1-x_0}} \leftarrow \text{IMP} \rightarrow \boxed{m_{sec} = \frac{f(x_0+h)-f(x_0)}{h}}$$

→ x0 & x1

AVERAGE VELOCITY

If P(x0,y0) is ~~of~~ a point on the graph of a function f, the tangent line to the graph of f at P is defined to be the through P with slope

$$\text{IMP} \rightarrow \boxed{m_{tan} = \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}}$$

where $h = x_1 - x_0$

LIMP $h \to 0$

The tangent line at P(x0,y0) is called the tangent line at x0 & so the point slope form of the equation of the tangent at x0 is

$$\boxed{y - y_0 = m_{tan}(x - x_0)} \rightarrow \text{IMP}$$

IMP →

The slope of a tangent line at any arbitrary point P(x,y) on the curve is given by

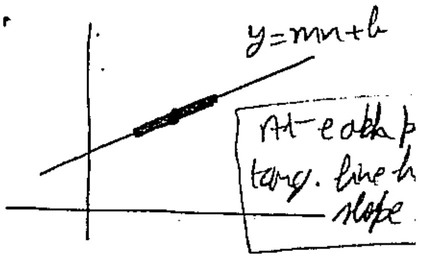
$$\boxed{m_{tan} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$$

↑ IMP

$\boxed{m_{tan} = m_{sec}\ Q \to P} \rightarrow$ IMP

* At each point on a line $\boxed{y = mx+b}$ the tangent line coincides with the line itself and thus has the slope m. Therefore if $f(x) = mx+b$, then $f'(x) = m$ for all x.

IMP



$$f'(x) = \lim_{h\to 0} \frac{m(x+h)+b - mx-b}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{mx}+mh\cancel{+b}-\cancel{mx}\cancel{-b}}{h} = \lim_{h\to 0}\frac{mh}{h} = \boxed{m}$$

AVERAGE & INSTANTANEOUS VELOCITIES

* The average velocity of an object moving in one direction along a line

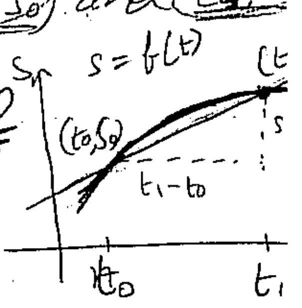
$\boxed{\text{Average Velocity} = \text{Distance travelled} / \text{Time elapsed}}$ → IMP

It may speed up or slow down → IMP

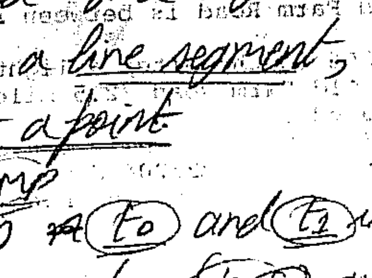
If a lapse of time (Average Velocity) is pictured by a length along a lin then an interval (of time) is represented by a line segment, whereas a instant (Instantaneous velocity) corresponds to a point.

IMP

The average velocity of the car between times ~~t~~ t_0 and t_1 is represe geometrically by the slope of the secant line connecting (t_0, s_0) and $(t_1, s$ on the position vs time curve. $\boxed{\text{Ave Velocity} = m_{sec}}$ → IMP



The instantaneous velocity of the car at time t_0 is represented by the slope of the tangent line at (t_0, s_0) on the position vs time curve. $\boxed{\text{Inst. Velocity} = m_{tan}}$



* $\boxed{(x+y)^3 = x^3+3x^2y+3xy^2+y^3}$
* $\boxed{x^3+y^3 = (x+y)(x^2-xy+y^2)}$
* $\boxed{(x-y)^3 = x^3-3x^2y+3xy^2-y^3}$
* $\boxed{x^3-y^3 = (x-y)(x^2+xy+y^2)}$

IMP

IMP

If (x_0, y_0) and (x_1, y_1) are points on the graph of $y = f(x)$, then

IMP

$$\boxed{m_{sec} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}}$$

to be the – IMP

Average rate of change of y w/ respect to x over the Interval $[x_0, x_1]$

If (x_0, y_0) is a point on the graph of $y = f(x)$, then the Instantane

rate of change of y w/ respect to x at x_0 → IMP

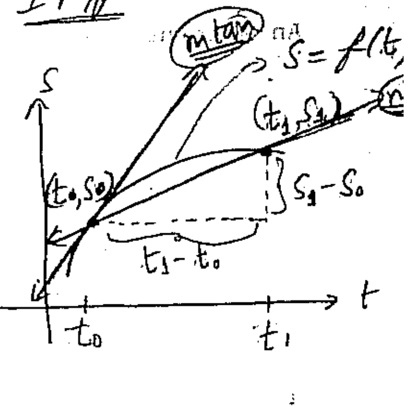
$$\boxed{m_{tan} = f'(x_0)} \quad [Q \to P]$$

IMP

$$\boxed{m_{tan} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}}$$

IMP

IMP



The function f' is defined as

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x_0+h) - f(x)}{h}}$$

$\boxed{h = x_1 - x_0}$

IMP

is the called the first derivative ~~of y~~ with respect to x to the func f. The Domain of f' consists of all x for which the limit exis

f' is the function whose value at x is the slope of the tangent line to $y - f(x)$ at x.

If $y = f(x)$, then f' is the function whose value at x is the instantaneous ~~vele~~ rate of change of y with respect to x at point → IMP (Instantaneous rate change)

Average change of y w/ respect to x

Average velocity

m_{sec}

Average rate of change = $\frac{f(x+h) - f(x)}{h}$ (no lim)

m_{tan}

Instantaneous velocity

$f'(n)$

Instantaneous rate of change of y w/ respect to n.

Instantaneous rate of change of volume w/ respect to r.

$\boxed{f'(n)}$ → ~~frist~~ first derivative of $f(n)$ w/ respect to n.

$\boxed{y'}$ * $\boxed{f'}$ * $\boxed{\frac{dy}{dn}}$ → derivative of y w/ respect to n

$\boxed{D_n}$ → Derivative of function w/ respect to n

$\boxed{\frac{dn^2}{dn}}$ → Polynomial function

$$\boxed{\left.\frac{d(n^2)}{dn}\right|_{n=2} = f'(2)}$$

Binomial Expansion → IMP

$$\boxed{(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + \binom{n}{n}}$$

* To find the particular term

IMP $\left\{ \boxed{\binom{\text{power}}{\text{term}-1}(n)^{\text{power}-(\text{term}-1)}(y)^{\text{term}-1}} \right.$

* PASCAL'S TRIANGLE → IMP MOS

$$* \quad \boxed{(n+y)^4 = n^4 + \frac{4}{1}n^3y + \frac{4\times3}{1\times2}n^2y^2 + \frac{4\times3\times2}{1\times2\times3}ny^3 + y^4}$$

EXISTENCE OF DERIVATIVES

* A function $f(n)$ has a differential at n or has a derivative at n if ~~the functi~~ n is not a—

a) Corner of the graph
b) Vertical tangent
c) points of discontinuity

IMP

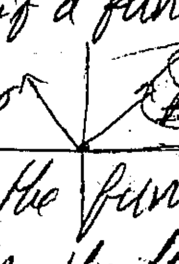
$(n-y)^3 = n^3 - 3n^2y + 3ny^2$

If f is differentiable at a point x_0, then f is also continuous at x_0. But if a function is continuous at x_0, it is not necessarily differentiable at x_0.
not (a,b) but point x_0

IMP When there exists a tangent line at point P on a curve it has to be the same line coming in from both left & right sides

V.V.V. IMP

* To find out if a function is continu
at a particular point take the derivative of the function at the point & if it exists, then the function is continuous



* $f(x) = |x|$
Is there $f'(0)$

$\Rightarrow f(x) = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$

IMP PROB

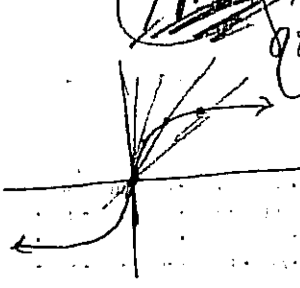
$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} \Rightarrow \lim_{h \to 0} \frac{|h|}{h}$$

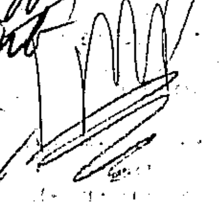
IMP
In this case
come in from
LEFT & RIGHT SIDE

(i) $\lim_{h \to 0^+} \frac{|h|}{h} = 1$ (ii) $\lim_{h \to 0^-} \frac{|h|}{h} = -1$

$\therefore$ Is a **DNE**

IMP Has to be continuous & be differentiable at a point





* $f(x) = x^{1/3}$
Does $f'(0)$ exist?

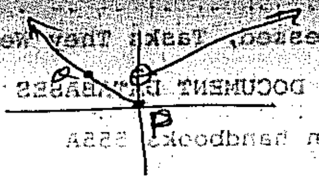
$\Rightarrow \lim_{h \to 0} \frac{\sqrt[3]{0+h} - 0}{h}$

$= \lim_{h \to 0} \frac{\sqrt[3]{h}}{h} \Rightarrow \lim_{h \to 0} \frac{1}{h^{2/3}}$

IMP
Again look at both

(i) $\lim_{h \to 0^-} \frac{1}{h^{2/3}} = \lim_{h \to 0^+} \frac{1}{h^{2/3}} = +\infty$

$$f(x) = \begin{cases} x^2, & x \le 0 \\ x+1, & x > 0 \end{cases}$$



there is no differential at f'(0)

IMP PROB

* $$f(x) = \begin{cases} x^2, & x \le 0 \\ x, & x > 0 \end{cases}$$

TMP

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$ { again look at both

(i) $\lim_{h \to 0} \frac{h-0}{h} = \lim_{h \to 0^+} 1 = 1$ (ii) $\lim_{h \to 0} \frac{h^2-0}{h} = \lim_{h \to 0^-} h = 0$

= DNE

TECHNIQUES OF DIFFERENTIATION

* If f is a continuous function, say f(x) = c for all x, then $f'(x) = 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c-c}{h} = \lim_{h \to 0} 0 = 0$$

$$\therefore \boxed{\frac{d}{dx}[c] = 0}$$

* If n is a positive integer, then

$$\boxed{\frac{d}{dx}[x^n] = nx^{n-1}}$$

* Proof: $\boxed{\frac{d}{dx}[x^n] = nx^{n-1}}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \Rightarrow \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h\to 0}\frac{\left(x^n + \frac{n}{1}x^{n-1}h + \frac{n\times n-1}{1\cdot 2}x^{n-2}h^2 + \ldots + h^n\right) - x^n}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{x^n} + nx^{n-1}h + \frac{n\times n-1}{2}x^{n-2}h^2 + \ldots + h^n - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0} = \frac{\cancel{h}\left(nx^{n-1} + \frac{n\cdot n-1}{2}x^{n-2}h + \ldots + h^{n-1}\right)}{\cancel{h}}$$

$$= \lim_{h\to 0} nx^{n-1} + \frac{n\cdot n-1}{2}x^{n-2}h + \ldots + h^{n-1} = \underline{\underline{nx^{n-1}}}$$

* Let c be a constant. If f is differentiable at x, so is cf, and $\boxed{\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]}$ or $\boxed{(cf)' = c}$

Proof: $\lim_{h\to 0}\frac{cf(x+h)-cf(x)}{h}$

$$= \lim_{h\to 0} c\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$\Rightarrow c\left[\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\right]$$ {a constant can be p out from a limit

* The slope of a Horizontal tangent line is =0 → IMP

* The slope of a vertical tangent is undefined

If f and g are differentiable at x, then so is f+g a

$$\boxed{\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]}$$

$$\boxed{(f+g)' = f'+g'}$$

$$\boxed{\text{V.V.V. IMP}}$$

Proof: $\lim\limits_{h\to 0} \frac{[f(x+h)+g(x+h)]-[f(x)-g(x)]}{h}$

$\Rightarrow \lim\limits_{h\to 0} \frac{f(x+h)+g(x+h)-[f(x)-g(x)]}{h}$

$\Rightarrow \lim\limits_{h\to 0} \frac{f(x+h)-f(x)+g(x+h)-g(x)}{h}$

The limit of a sum is the sum of the limits $\Rightarrow \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim\limits_{h\to 0} \frac{g(x+h)-g(x)}{h}$

$\Rightarrow \underline{\lim\limits_{h\to 0} f(x) + \lim\limits_{h\to 0} g(x)}$

$$\boxed{\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]}$$

$$\boxed{(f-g)' = f'-g'}$$

$$\boxed{\text{V.V.V. IMP}}$$

Proof: $\lim\limits_{h\to 0} \frac{[f(x+h)-g(x+h)]+[f(x)-g(x)]}{h}$

$\Rightarrow \lim\limits_{h\to 0} \frac{f(x+h)-f(x)}{h} - \frac{g(x+h)-g(x)}{h}$

$\Rightarrow \underline{\lim\limits_{h\to 0} f(x) - \lim\limits_{h\to 0} g(x)}$

---

* $y = (3x^2+1)^2 = \underline{y = 9x^4+6x^2+1}$ ✓ $\Rightarrow 36x^3+12x \rightarrow$

NOT $\Rightarrow (6x)^2$ ✗

*if f and g are differentiable at x, then so is the product $f \times g$, and

V.V.V. IMP

$$\boxed{\frac{d}{dx}[f(x) \times g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]}$$

$$\boxed{(f \cdot g)' = f \cdot g' + g \cdot f'}$$

Proof

$$\lim_{h \to 0} \frac{f(x+h) \times g(x+h) - f(x) \times g(x)}{h}$$

add & subtract $f(x+h) \cdot g(x)$ in the numerator

$$\lim_{h \to 0} \frac{f(x+h)\, g(x+h) - \cancel{f(x+h) \cdot g(x)} + \cancel{f(x+h) \cdot g(x)} - f(x) g(x)}{h}$$

$$= \lim_{h \to 0} f(x+h)\left(\frac{g(x+h) - g(x)}{h}\right) + g(x) \times \left(\frac{f(x+h) - f}{h}\right.$$

$$= \lim_{h \to 0} f(x+h) \cdot \lim_{h \to 0} g(x) + \lim_{h \to 0} g(x) \times \lim_{h \to 0} \frac{f(x+h) - f}{dh}$$

$$\Rightarrow \lim_{h \to 0} f(x+h) \times \lim_{h \to 0} g(x) + \lim_{h \to 0} g(x) \times \lim_{h \to 0} f(x)$$

$\lim_{h \to 0} g(x) = g(x)$ & the same for others

$$\Rightarrow f(x+h) \times g(x) + g(x) \times f(x)$$

$$\frac{d}{dx}[f(x) \cdot g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

It is not true in general that $(f \cdot g)' = f' \cdot g'$ but

$$(f \cdot g)' = f g' + g f'$$

$$\boxed{\frac{d}{dn}[f(n)/g(n)] = \frac{g(n)\frac{d}{dn}[f(n)] - f(n)\frac{d}{dn}[g(n)]}{[g(n)]^2}}$$

$$\boxed{(f/g)' = \frac{gf' - fg'}{g^2}}$$

V.V.V. IMP

$$\boxed{\frac{d}{dn}\left[\frac{1}{g(n)}\right] = \frac{\frac{d}{dn}[g(n)]}{[g(n)]^2}} \quad a \quad \boxed{\left(\frac{1}{g}\right)' = \frac{g'}{g^2}}$$

V.V.V. IMP

$$\boxed{\frac{d}{dn}[\sqrt{n}] = \frac{1}{2\sqrt{n}}}$$

IMP

Proof: $\frac{d}{dn}[n^{1/2}] = \cancel{d}\,\frac{1}{2}n^{-1/2} = \frac{1}{2\sqrt{n}}$

$$\boxed{f^{(n)}(n) = \frac{d^n}{dn^{(n)}}[f(n)]}$$ => the nth derivative of f with respect to n

If f is continuous at n_0 and that = $\lim_{n \to n_0^+} f'(n)$ and $\lim_{n \to n_0^-} f'(n)$ exists, then f is differentiable at n_0 if and only if these limits are equall.

$$\boxed{f'(n_0) = \lim_{n \to n_0^+} f'(n) = \lim_{n \to n_0^-} f'(n)}$$

IMP

LOGIC

Function f should be continuous → IMP

DERIVATIVES OF TRIG FUNCTIONS

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$1 - 2\sin^2\theta$

$2\cos^2\theta - 1$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\sin \frac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\cos \frac{1}{2}\theta = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\tan \frac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2$

$$* \boxed{\frac{d}{dy}[\sin x] = \cos x} \longrightarrow ①$$

Proof:

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin x\cos h + \sin h\cos x - \sin x}{h}$$

$$= \lim_{h\to 0}\left[\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right]$$

$$\Rightarrow \lim_{h\to 0}\left[-\sin x\left(\frac{1-\cos h}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right]$$

$$\Rightarrow \lim_{h\to 0} -\sin x(0) + \cos(x)(1)]$$

$$\lim_{h\to 0} \cos x = \underline{\cos x}$$

Page 2

:ibarkeep!conor (or conor.rafferty@att.com).

Next Trip Committee Meeting

next trip committee meeting is scheduled for November 11 in Summit.
e details will be sent to people on the volunteer list in November.
you are interested in volunteering to help with publicity, leading trips,
ers, location representatives, etc., and are not currently on the
unteer list, send mail to Mary Hesselgrave
uxlm!mth or hesselgrave@att.com).

* $\boxed{\frac{d}{dx}[\cos x] = -\sin x} \rightarrow$ ② Proof: $\lim_{h\to0} \frac{\cos(x+h)-\cos x}{h}$

$\lim_{h\to0} \frac{\cos x\cos h - \sin x\sin h - \cos x}{h} \Rightarrow \lim_{h\to0} \cos x\frac{(\cos h - 1)}{h} - \sin x\left(\frac{\sin h}{h}\right.$

$\Rightarrow \lim_{h\to0}\left[-\cos x\left(\frac{1-\cos h}{h}\right) - \sin x\left(\frac{\sin h}{h}\right)\right]$

$\lim_{h\to0}\left[\cos x\,(0) - \sin x\,(1)\right]$

$\Rightarrow \lim_{h\to0} -\sin x = \underline{-\sin x}$

* $\boxed{\frac{d}{dx}[\tan x] = \sec^2 x} \rightarrow$ ③ Proof: $\frac{d}{dx}\left[\frac{\sin x}{\cos x}\right]$

$\frac{\cos x\frac{d}{dx}[\sin x] - \sin x\frac{d}{dx}[\cos x]}{\cos^2 x}$

$\frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$\frac{1}{\cos^2 x} = \underline{\sec^2 x}$

④ * $\boxed{\frac{d}{dx}[\cot x] = -\csc^2 x}$

⑤ * $\boxed{\frac{d}{dx}[\sec x] = \sec x\tan x}$

* $\boxed{\frac{d}{dx}[\csc x] = -\csc x\cot x} \rightarrow$ ⑥

* $\boxed{\frac{d}{dn}[\csc n] = -\csc n \cot n}$

~~lim~~ $\frac{d}{dn}\left[\frac{1}{\sin n}\right] = \lim_{h\to 0} \frac{\sin n(0) - (1)(\cos n)}{\sin^2 n}$

$= \lim_{h\to 0} \frac{-\cos n}{\sin^2 n} = -\frac{\cos n}{\sin n} \times \frac{1}{\sin n} \Rightarrow \lim_{h\to 0} \underline{-\cot n \csc}$

$\Rightarrow \underline{\underline{-\cot n \csc n}}$

* $\boxed{\frac{d}{dn}[\sec n] = \sec n \tan n}$

$\Rightarrow \frac{d}{dn}\left[\frac{1}{\cos n}\right] = \lim_{h\to 0} \frac{\cos n(0) - (1)(-\sin n)}{\cos^2 n}$

$\Rightarrow \lim_{h\to 0} \frac{+\sin n}{\cos^2 n} = \frac{\sin n}{\cos n} \times \frac{1}{\cos n} \Rightarrow \lim_{h\to 0} \tan n \sec$

$\Rightarrow \underline{\sec n \tan n}$

* $\boxed{\frac{d}{dn}[\cot n] = -\csc^2 n}$ $\quad \frac{d}{dn}\left[\frac{\cos n}{\sin n}\right] = \frac{\sin n(-\sin n) - \cos n(\cos}{\sin^2 n}$

$= \frac{-\sin^2 n - \cos^2 n}{\sin^2 n} = \frac{-1(\sin^2 n + \cos^2 n)}{\sin^2 n} = \frac{-1(1)}{\sin^2 n}$

$= \cancel{\csc} = \underline{-\csc^2 n}$

* Given a trig function, the points where the graph of f has a horizontal tangent line are the points wh the slope of the function or the differential of the function $\boxed{=0}$ $\boxed{\text{IMP}}$

* For a function to be differentiable, it has to be continuous & for it to be continuous there should be no breaks or undefined pts

THE CHAIN RULE

If we know the derivatives of f & g, we can use this information to find the derivative of f∘g.

$y = (f \circ g)(x) = f(g(x))$ and $u = g(x)$

so $y = f(u)$ ∴ $\frac{dy}{du} = f'(u)$ & $\frac{du}{dx} = g'(x)$

to find the unknown derivative

$$\frac{dy}{dx} = \frac{d}{dx}[f(g(x))]$$

We want to find the unknown rates of change using the known rates of change and we know that the unknown rates of change multiply.

Page 1

```
n: !march
e: Wed Aug 19 16:58:23 EDT 1992
ne: 908-949-3072
ject: CSC Meeting
 !sgbali
 !march
 mtdca!wgn
 !vsh
 !tlr
 mtdcr!vbl
 attmail!nshaer
tent-Length: 2690
```

i,

(Tom's and our group) do not have to attend these meetings. Bill Newcomb
I talked about this. He explained that the CSC activities are in response
management's request to reduce computing costs at Middletown. They are
ing from VAXes to SUN-based servers. As you know we have already cut our
ts here in Holmdel through our own initiative by migrating from the
nframe-based UNIX logins to the new StarLAN/DOS based computing service now
ng offered by the Holmdel Computation Center.

Bill and I discussed, there are no issues related to the CSC activities
t we need to be involved in. Bill and I will contact each other on an as
ded basis to deal with any issues common to our two user communities.

rc Hornby

If g is differentiable at point x and f is differentiable at the point g(x), then the composition f∘g is differentiable at the point x, if

$$y = f(g(x)) \; \& \; u = g(x)$$

→ one substitution

then $$\boxed{\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}}$$ — IMP

* There could also be more than one substitution so-

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} \times \frac{dt}{dx}} \rightarrow \text{IMP}$$

* There are two other ways of writing the derivative of a ~~fn~~ composition function. →

$$\boxed{\frac{dy}{dx} = \frac{d}{dx}[f(g(x))]} \; \boxed{\frac{dy}{du} = f'(u) = f'[g(x)]}$$

$$\& \; \boxed{\frac{du}{dx} = g'(x)}$$

V.V.V. IMP

$$\therefore \Rightarrow \boxed{\frac{d}{dx}[f(g(x))] = f'[\underset{u}{g(x)}] \cdot g'(x)} \text{ — IMP}$$

$$\Rightarrow \boxed{\frac{d}{dx}[f(u)] = f'(u)\,\frac{du}{dx}} \text{ — IMP}$$

P.T.O

IMP * The derivative of $f(g(x))$ is the derivative of the outside function evaluated at the inside function times the derivative of the inside function.

~~* $y = (5x^2+2)^9$~~

* To find ~~#~~ which function is outer and which is inner, plug in ~~for~~ a value for x & determine which function needs to be done first (Inner). IMP

* $y = (5x^2+2)^9$

$y' = 9(5x^2+2)^8\,(10x)$

$y' = 90x\,(5x^2+2)^8$

* $y = (7x^2+2x+1)^4$

$y' = 4(7x^2+2x+1)^3(14x+$

* $y = \tan 2x$ prod. rule

$y' = \sec^2 2x\,(2(1)+x$

$= \sec^2 x\,(2)$

$= 2\sec^2 x$

* $y = \cos(2x^2+x)$

$y' = -\sin(2x^2+x)\,(4x+1)$

```
------------ Begin Forwarded Message ------------
sage-Version: 2
: march
m: !sgbali
e: Wed Aug 19 14:11:19 EDT 1992
-Content-ID: <PMX-LAN-2.2.1-******-hostar-sgbali-40>
-of-Header:
il-Version: 2
Message-ID: <winPMXSTAR-2.2.1b-sgbali-XYXYXYXYXY-32>
: !march
-of-Protocol:
tent-Type: Text
tent-Length: 1548

rc,
uld you please take care of it.
anks
```

* $y = \tan^{12} n = (\tan n)^{12}$

$y' = 12(\tan n)^{11} \times (\sec^2 n)$

$= 12(\tan n)^{11}(\sec^2 n)$

* $y = \sin^{12} n^3 = (\sin n^3)^{12}$

$y' = 12(\sin n^3)^{11} \cdot (\sin n^3)'$

$= 12(\sin n^3)^{11} \times (\cos(n^3) \times (3n^2)')$

$= 12(\sin n^3)^{11} \times \cos(n^3) \times 3n^2$

$= 36n^2(\sin n^3)^{11} \cos n^3$

* $$y = \frac{(n^2+3n)^3}{n^2-1}$$

$$y' = \frac{(n^2-1)\{(n^2+3n)^3\}' - (n^2+3n)^3\,(n^2-1)'}{(n^2-1)^2}$$

$$y' = \frac{(n^2-1)\{3(n^2+3n)^2(2n+3)\} - (n^2+3n)^3(2n)}{(n^2-1)^2}$$

$$= \frac{(n^2-1)\,3(n^2+3n)^2(2n+3) - (n^2+3n)^3(2n)}{(n^2-1)^2}$$

$$\frac{-(n^2+3n)^2\left[-(n^2-1)(2n+3) + (n^2+3n)(2n)\right]}{(n^2-1)^2}$$

$$* \; f(n) = \frac{4}{(3n^2-2n+1)^3} = 4(3n^2-2n+1)^{-3}$$

$$y' = 4\{(3n^2-2n+1)^{-3}\}' + (3n^2-2n+1)^{-3}(4)'$$

$$= 4(-3(3n^2-2n+1)^{-4}(6n-2)) + 0$$

$$= \cancel{4} -12(3n^2-2n+1)^{-4}(6n-2)$$

$$* \; f(n) = n^2\sqrt{5-n^2} = n^2(5-n^2)^{1/2}$$

$$y' = n^2\{(5-n^2)^{1/2}\}' + (5-n^2)^{1/2}(n^2)'$$

$$+ (5-n^2)^{+1/2}[2n)$$

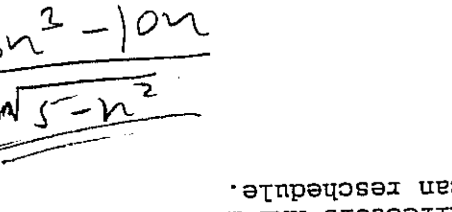
$$= n^2\left(\tfrac{1}{2}(5-n^2)^{-1/2}(-2n)\right) + (5-n^2)^{1/2}(2n)$$

$$= n^2\tfrac{1}{2}(5-n^2)^{-1/2}(-\cancel{2}n) + (5-n^2)^{1/2}(2n)$$

$$= (5-n^2)^{-1/2}[n^3 - 10n + 2n^3]$$

$$\Rightarrow \frac{3n^3-10n}{\sqrt{5-n^2}}$$

NORMAL = ? ~~-ive Reciprocal~~ of mtan →



---------- Begin Forwarded Message ------------
age-Version: 2
:mtdca!wgn
: Fri Aug 14 17:38 EDT 1992
ived: from mtdca by hostar.ho.att.com; Fri, 14 Aug 1992 17:41 EDT
of-Header:
l-Version: 2
ect: Next CSC meeting (may be MANDATORY)
mtdca!org=140:org=45351:all=y
of-Protocol:
ent-Type: text
ent-Length: 1113

o everyone;

next Computer Steering Committee otherwise known as CSC is set
Tues.. the 18th at 10:00 am in 1p-266a.

is a mandatory meeting of all Dept. representatives. Things, and I
lots of things, are happen`en SOON and all MUST be informed.
oze and you lose...and we do not want to lose.

he agenda for the next meeting is some more details on the Vax shutdowns.
THE VAXES WILL BE SHUTDOWN. NO ONE IN THEIR RIGHT MIND IS GOING TO STAY
THE VAX AND BE CAUGHT PAYING THE BILL. There will be more detailed message
ut this shortly. Also, some data on printer alternatives, some more
work status, some details on the SDE migration and we will have
usual general discussions. ALL are invited, BUT EACH DEPARTMENT MUST
E A REPRESENTATIVE. The department heads, VPs, and directors ARE EXPECTING
s. If you can not make it, send an alternate or we can reschedule.

e to the the next meeting:

Tuesday 10:00
1P-266A

comments or suggestions...send me email (or call)
liam Newcomb (Billn)
2D-221
ca!wgn
-957-3577

IMPLICIT DIFFERENTIATION

* We can differentiate both sides of an eqn before solving for y in terms of x, treating y as a (temporarily unspecified) differentiable function of x.

* $xy = 1$

$x\frac{dy}{dx} + y = 0$

$\frac{dy}{dx} = -\frac{y}{x} \Rightarrow -\frac{1}{x^2}$

V.V. IMP [* It is useful when it is inconvenient or impossible to solve explicitly for y in terms of x.

* $5y^2 + \sin y = x^2$

$5\cdot 2y\frac{dy}{dx} + \cos y\frac{dy}{dx} = 2x$

$\frac{dy}{dx}(10y + \cos y) = 2x$

$\frac{dy}{dx} = \frac{2x}{10y + \cos y}$

V.V. IMP

* In order to obtain a formula involving x alone, we would have to solve the original equation for y in terms of x & substitute it, but it is impossible to do it

* Slope of the tangent line at (4,0), to the graph of

$7y^4 + x^3y + x = 4$ [(4,0) lies on the graph]

$28y^3\frac{dy}{dx} + 3x^2y + x^3\frac{dy}{dx} + 1 = 0$

$\frac{dy}{dx}(28y^3 + x^3) = -3x^2y - 1$

$\frac{dy}{dx} = \frac{-3x^2 0 - 1}{28y^3 + x^3} = \frac{-1}{64}$

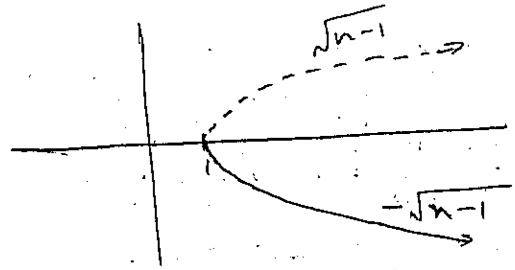
* Slope of the tangent line at $(2,-1)$ to $y^2 - x + 1 = 0$

$$y' = 2y\frac{dy}{dx} - 1 = 0$$

$$\frac{dy}{dx} = \frac{1}{2y} \quad \text{at } (2,-1) = \underline{\underline{\frac{-1}{2}}}$$

$\boxed{\text{Alternative Solution}}$: $y^2 - x + 1 = 0$

~~$y = \sqrt{x+1}$~~ $y = \pm\sqrt{x-1}$



$$\frac{dy}{dx}\left[-\sqrt{x-1}\right] =$$

$$= -\frac{1}{2}(x-1)^{-1/2}(1)$$

$$= \frac{-1}{2\sqrt{x-1}} \quad \text{at } x = 2$$

$$= \underline{\underline{\frac{[illegible]}{2}}}$$

* $\frac{d^2y}{d^2x}\left[4x^2 - 2y^2 = 9\right]$

$$= 4x^2 - 2y^2 - 9 = 0$$

$$4(2x) - 2\left(2y\frac{dy}{dx}\right) - 0 = 0$$

$$8x - 4y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{+8x}{+4y} = +\frac{2x}{y}$$

$$\frac{d}{dx}\left[+\frac{2x}{y}\right] = \frac{y(+2) - +2x\left(\frac{dy}{dx}\right)}{y^2} = \frac{+2y - 2x\frac{dy}{dx}}{y^2}$$

$$= \frac{+2y - 2x\left(\frac{+2x}{y}\right)}{y^2} = \frac{+2y - \frac{4x^2}{y}}{} = \frac{+2y^2 - 4x^2}{y^3}$$

$$\rightarrow \frac{2y^2 - 4x^2}{y^3}$$

MOST IMP ← $\boxed{\text{Substitute from Orig. Eq.}}$

$$= \underline{\underline{-\frac{9}{y^3}}}$$

OVER

* If u is a differentiable function of n, and r is a rational number then the chain rule yields ~~the~~ –

$$\boxed{\frac{d}{dn}[u^{r}] = ru^{r-1}\cdot\frac{du}{dn}} \rightarrow \text{IMP}$$

* when differentiating implicitly it is assumed tha y represents a differentiable function of n. → IMP
* Can Use the alternative form if the coordinates are given

Δ NOTATION DIFFERENTIALS

* The final value minus the initial value of a variable is in increment.
* the increment in n is Δn

$$\boxed{\Delta n = n_1 - n_0}$$

& $\boxed{\Delta y = y_1 - y_0 \text{ or } f(n_1) - f(n_0)}$

* Increments can be either positive or negative depending on the relative final & initial positions of the points

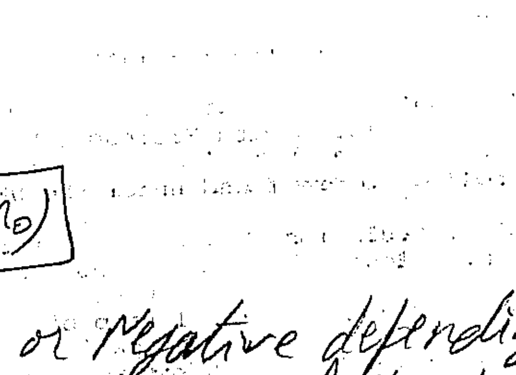
$$\Delta y = f(n_0+h) - f(n_0)$$

& we know that $h \approx$

$$\boxed{\Delta y = f(n_0 + \Delta n) - f(n_0)}$$

IMP

$\Delta n \approx h$



* We know that $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

or $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ or $\boxed{\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}}$

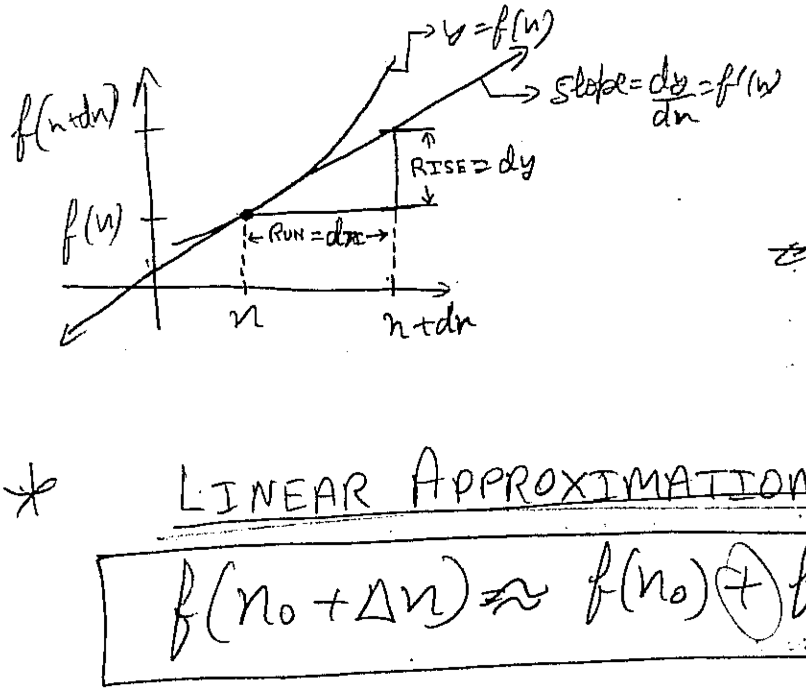
$$\therefore \boxed{\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}} \rightarrow \text{IMP}$$

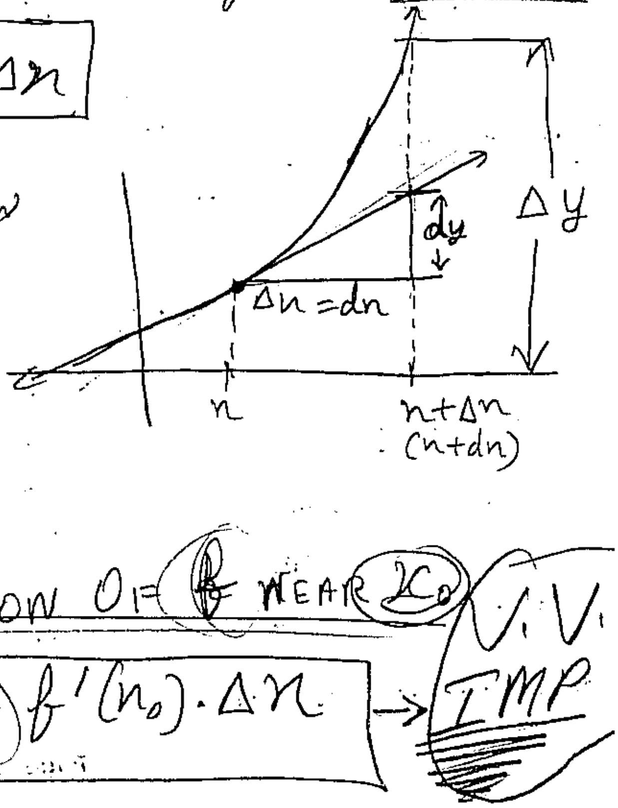
* We also know that $\frac{dy}{dx} = f'(x) = m_{tan}$, where dy and dx are <u>differentials</u>.

$\therefore$ We can rewrite dy as $\boxed{dy = f'(x) \times dx}$ ← IMP

where dy corresponds to the <u>RISE</u> of a <u>tangent</u> and dx corresponds to the <u>RUN</u> of the <u>tangent li</u>

+ In most cases $\boxed{dx = \Delta x}$





* <u>LINEAR APPROXIMATION OF f NEAR x_0</u> V.V. IMP

$$\boxed{f(x_0 + \Delta x) \approx f(x_0) + f'(x_0) \cdot \Delta x} \rightarrow \text{IMP}$$

APPLICATION OF DIFFERENTIATION

4. APPLICATIONS OF DIFFERENTIATION

RELATED RATES

* $xy = 2$ w/ respect to x

$$\frac{dy}{dx} = \frac{-y}{x}$$

* $xy = 2$ w/ respect to y

$$x + \frac{dx}{dy} y = 0$$

$$\therefore \frac{dx}{dy} = \frac{-x}{y}$$

* w/ respect to t

$$xy = 2$$

$$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$$

$$\therefore \frac{dy}{dt} = \frac{-y}{x}\frac{dx}{dt}$$

↳ change of y with respect to t.

* Rate is rate of change with respect to time.
* Fast ≈ Rate

* One rate ($\frac{dy}{dt}$) is dependent upon another rate ($\frac{d}{d}$

Step 1: Draw a figure & label the quantities that vary

Step 2: Find an equation relating the quantity an the unknown rate of change to quantities wh rate of change are known.

IMP

Step 3: Differentiate both sides of this equation with respect to time and solve for the derivative that will give the unknown rate of change.

IMP

Step 4: Evaluate this derivative at the appropria point.

Impt

put in a minus sign at the rate whenever it is decreasing.

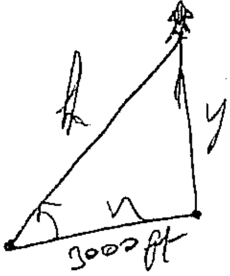
$\frac{dr}{dt} = 2 ft/sec \quad \frac{da}{dt} = ? \Big|_{r=60}$

$a = \pi r^2$

$\frac{da}{dt} = \pi 2r \frac{dr}{do}$

$\frac{da}{dt} = 2 \times 60 \times \pi \times 2 = \underline{240\pi} ft^2/sec$

3, 4, 5
5, 12, 13
8, 15, 17
7, 24, 25
6, 8, 10



$\frac{dy}{dt} = 880 ft/sec \quad \frac{dh}{dt} = ? \Big|_{\cancel{y}=4000 ft}$

$(3000)^2 + y^2 = h^2$

$0 + 2y\frac{dy}{dt} = 2h\frac{dh}{dt}$

Is a 3, 4, 5 △
∴ $\cancel{y}h = 5000$

$\therefore \frac{dh}{dt} = \frac{\cancel{2}y \times 880}{\cancel{2}h} = \frac{880y}{h}$

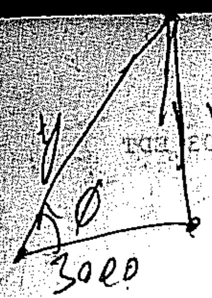
$\therefore \frac{\overset{176}{\cancel{880}} \times 4000}{\underset{5}{\cancel{5000}}} = \underline{704 ft/sec}$

```
---------- Begin Original Message ----------
sage-Version: 2
m: hoqax!jeromes
e: Sat Oct 10 11:06 EDT 1992
eived: from hoqax by hostar.ho.att.com; Sat, 10 Oct 1992 11:03 EDT
-of-Header:
il-Version: 2
ject: telecomm bmkg group
 kpd
 !sgball
-of-Protocol:
tent-Type: text
tent-Length: 167

i, KP,

 you tell me the name of the telecommunications
chmarking consortium that you referred to?
 like to be more aware of them.

nks,
ome.
```



$$\frac{dx}{dt} = 880 \text{ ft/sec}$$

$$\frac{d\theta}{dt} = ?\Big|_{x=4000}$$

$$\tan\theta = \frac{x}{3000}$$

$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{3000}\times\frac{dx}{dt}$$

$$\therefore \frac{d\theta}{dt} = \frac{1}{3000}\times\frac{dx}{dt}\times\cos^2\theta$$

$$\frac{d\theta}{dt} = \frac{1}{3000}\times 880\times\cos^2\theta$$

~~$\cos^2\theta = \frac{3}{3520}\times 880\times\left(\frac{3}{5}\right)^2$~~

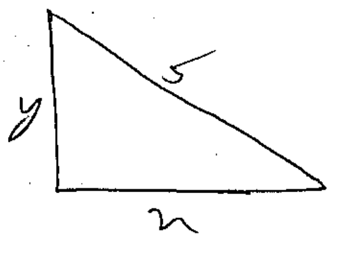
$$\frac{d\theta}{dt} = = \frac{1}{3000}\times 880\times\left(\frac{3}{5}\right)^2$$

$$\therefore \frac{d\theta}{dt} = \cancel{00}\ 0.11 \text{ rad/sec}$$

$$\therefore \cos\theta = \frac{3000}{y}$$

$$y = 5000$$

$$\therefore \cos\theta = \frac{3}{5}$$



$$\frac{dx}{dt} = 2 \text{ ft/sec} \quad \frac{dy}{dt} = ?\Big|_{x=4}$$

when x=4
then y = 3
as a 3,4,5 ∆

$$x^2+y^2 = 25$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\cancel{8\frac{dx}{dt}}\ 16 + 6\frac{dy}{dt} = 0$$

$$\therefore \frac{dy}{dt} = \frac{-16}{6} = \frac{-8}{3} \text{ ft/sec}$$

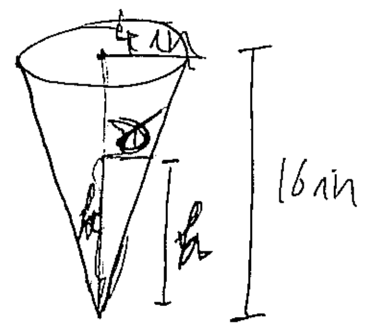
------------- Begin Original Message -------------
Message-Version: 2
From: hoqax!jeromes
Date: Sat Oct 10 11:06 EDT 1992
Received: from hoqax by hostar.ho.att.com; Sat, 10 Oct 1992 11:03 EDT
End-of-Header:
Email-Version: 2
Subject: telecomm bmkg group
To: kpd
To: !sgbali
End-of-Protocol:
Content-Type: text
Content-Length: 167

Shri, KP,

Can you tell me the name of the telecommunications
benchmarking consortium that you referred to?
I'd like to be more aware of them.

Thanks,
Jerome

~~IMP~~ ~~minus to indicate~~ decreasing



$$\frac{dv}{dt} = -2\ in^3/min$$

$$\frac{dh}{dt} = ?\ \Big|_{h=8}$$

IMP
⇓

$$\frac{4}{16} = \frac{r}{h}$$

$$h = 4r$$

$$\therefore r = h/4$$

$$V = \frac{1}{3}\pi r^2 h = V = \frac{1}{3}\pi \frac{h^2}{16} \times h$$

$$\frac{dv}{dt} = \frac{1}{3}\pi\left(\frac{h^2}{16}\frac{dh}{dt} + h\left(\frac{1}{16}2h\frac{dh}{dt}\right)\right)$$

$$\therefore \frac{dv}{dt} = \frac{1}{3}\pi\left(4\frac{dh}{dt} + 8\frac{dh}{dt}\right)$$

$$= -2 = 4\pi\frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = -\frac{1}{2\pi}\ in/min$$

$$V = \frac{4}{3}\pi r^3 \qquad \frac{dv}{dt} = 3 \qquad \frac{dd}{dt} = ? \;\Big/\; r = 1$$

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \qquad 2r = d$$

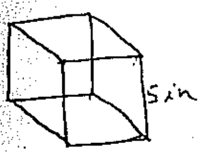
$$r = \frac{d}{2}$$

$$\frac{dv}{dt} = \frac{4}{3}\pi\left\{3\left(\frac{d}{2}\right)^2\left(\frac{1}{2}\frac{dd}{dt}\right)\right\}$$

$$3 = \frac{4}{3}\pi\left(3(1)\left(\frac{1}{2}\frac{dd}{dt}\right)\right\}$$

$$3 = \frac{4}{3}\pi \times \frac{3}{2}\frac{dd}{dt}$$

$$\frac{dd}{dt} = \frac{3}{2}\pi \text{ ft/sec}$$



$$V = n^3 \qquad \frac{dv}{dt} = 2 \qquad \frac{ds}{dt} = ?$$

$$S = 6n^2$$

$$n = 5$$

$$S = 6n^2$$

$$n = \sqrt{\frac{S}{6}} = \frac{\sqrt{6S}}{6} = \frac{(6S)^{1/2}}{6}$$

$$\therefore\ V = \left\{\frac{(6S)^{1/2}}{6}\right\}^3$$

$$\frac{dv}{dt} = 3\left(\frac{\sqrt{6S}}{36}\right)^2\left(\frac{1}{6}\left(\frac{1}{2}(6S)^{-1/2}\left(6\frac{ds}{dt}\right)\right)\right)$$

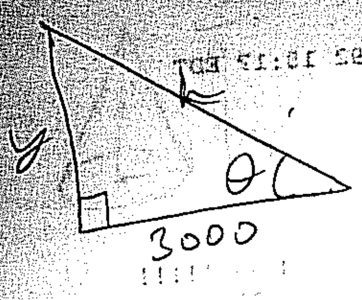
$$\frac{dv}{dt} = 3\left(\frac{6S}{366}\right)\left(\frac{1}{6}\left(\frac{1}{2\sqrt{6S}}\right)\left(6\frac{ds}{dt}\right)\right)$$

$$\frac{dv}{dt} = \frac{S}{2}\left(\frac{1}{12\sqrt{6S}}\right)\left(6\frac{ds}{dt}\right)$$

$$2 \times \frac{2}{S} \times 12\sqrt{6S} = 6\frac{ds}{dt}$$

$$\frac{ds}{dt} = \frac{4}{S} \times \cancel{12}^{2}\sqrt{6S} \times \frac{1}{\cancel{6}} = \frac{8\sqrt{6S}}{S}$$

$$\frac{ds}{dt} = \frac{4}{150} \times 2\sqrt{6} \times 150 = \frac{240}{150} = \frac{8}{5}\ \text{in}^2/\text{min}.$$



$\frac{dy}{dt} = 500$, $\frac{d\theta}{dt} = ?$ | $y = 3000$

$\tan\theta = \frac{y}{3000}$

$\sec^2\theta \frac{d\theta}{dt} \frac{1}{3000} \frac{dy}{dt}$

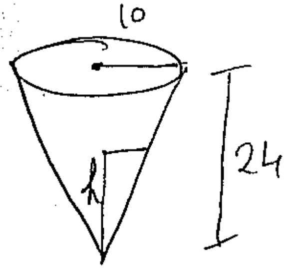
$\cos\theta = \frac{3000}{h}$

$h = 0.707$

$\frac{d\theta}{dt} = \frac{1}{3000} \times 500 \times \cos^2\theta$

$\frac{d\theta}{dt} = \frac{500}{3000} \times (0.707)^2$

$= 0.083$ rad/sec



$\frac{dV}{dt} = 20$ $\frac{dh}{dt} = ?$ | $h = 16$

$V = \frac{1}{3}\pi r^2 h$ & $r = \frac{10h}{24} = \frac{5h}{24}$

$\frac{dV}{dt} = \frac{1}{3}\pi \times \left(\frac{25}{144} h^2 \frac{dh}{dt} + h \times \frac{25}{144}\left(2h\frac{dh}{dt}\right)\right)$

$20 = \frac{1}{3}\pi\left(\frac{25}{144} \times 256 \frac{dh}{dt} + \frac{16 \times 25}{144}\left(2 \times 16 \frac{dh}{dt}\right)\right)$

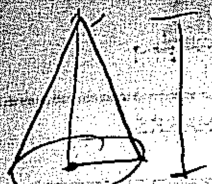
$20 = \frac{1}{3}\pi\left(\frac{6400}{144}\frac{dh}{dt} + \frac{12800}{144}\frac{dh}{dt}\right)$

$20 = \frac{6400}{432}\pi \frac{dh}{dt} + \frac{12800}{432}\pi \frac{dh}{dt}$

$20 = \frac{19200}{432}\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{9}{20}\pi$ in/sec

*



$\frac{dv}{dt} = 8\ ft^3/min$

$h = 2r$, ~~h~~ $r = \frac{h}{2}$

$\frac{dh}{dt} = ? \Big/ h = 6$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \times \frac{h^2}{4} \times h$

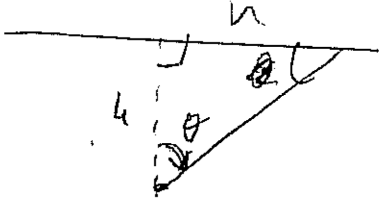
$\frac{dv}{dt} = \frac{1}{3}\pi\left(\frac{1}{4} \times 3h^2 \frac{dh}{dt}\right)$

$8 = \frac{1}{3}\pi\left(\frac{3h^2}{4}\frac{dh}{dt}\right)$

$8 = \frac{1}{3}\pi\left(\frac{3}{4} \times \overset{9}{\cancel{36}} \times \frac{dh}{dt}\right)$

$8 = 9\pi \frac{dh}{dt} \Rightarrow \therefore \frac{dh}{dt} = \frac{8}{9\pi}$ un/sec

*



$\frac{d\theta}{dt} = \frac{360}{10} = 36$ deg/sec

$\frac{dx}{dt} = ? \Big/ \theta = 45°$

$\tan\theta = \frac{x}{4}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{4}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{4}\frac{dx}{dt}\cos^2\theta$

We know that
$\cos 45° = \frac{1}{\sqrt{2}}$

$\frac{4\frac{d\theta}{dt}}{\cos^2\theta} = \frac{dx}{dt}$

$\Rightarrow \frac{dx}{dt} = 8 \times \frac{\pi}{5} = \frac{8\pi}{5}$ km/sec

INTERVALS OF INCREASE AND DECREASE & CONCAVITY

* Let f be defined ~~at~~ on an interval. Then:

a) f is increasing on the interval if for any points x_1 & x_2 in the intervals such that $x_1 < x_2$, we have $f(x_1) < f(x_2)$

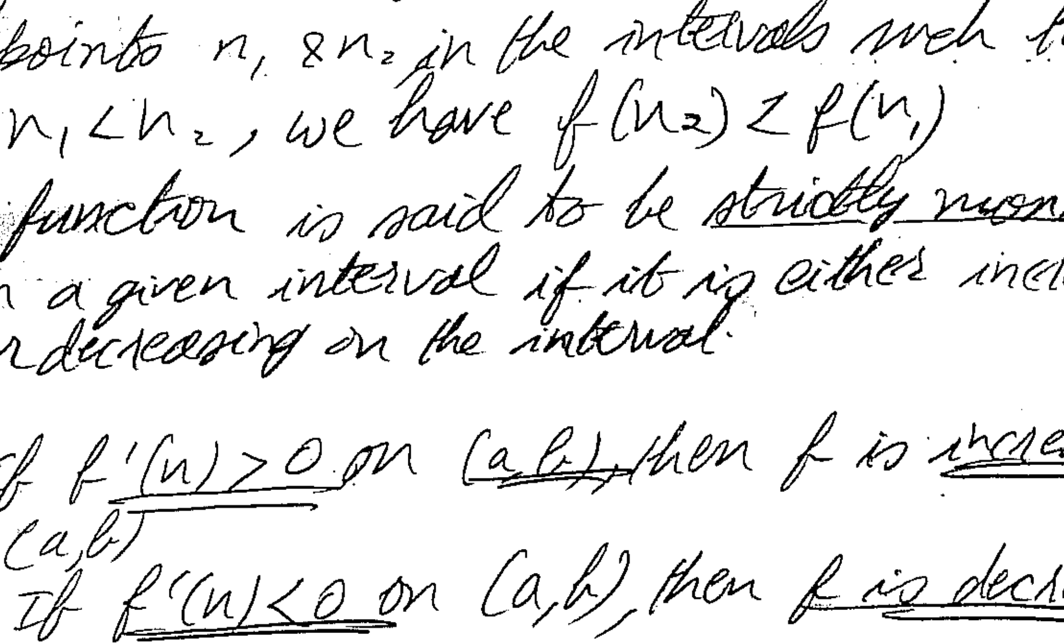
b) f is decreasing on the interval if for any points x_1 & x_2 in the intervals such that $x_1 < x_2$, we have $f(x_2) < f(x_1)$

* A function is said to be strictly monotone on a given interval if it is either increasing or decreasing on the interval

IMP

* a) If $f'(x) > 0$ on (a,b), then f is increasing on (a,b)

b) If $f'(x) < 0$ on (a,b), then f is decreasing on (a,b)



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|        |    |            |                   |     |     |;;;;V. L.          |
|        |    |            |                   |     |     |Davenport;;;;;T~C;;|
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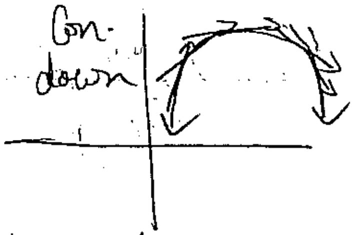
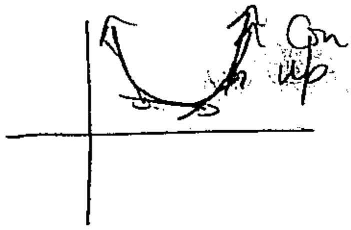
CONCAVITY





IMP

* a) f is concave up on (a,b) if f' is increasing on (a,b)

b) f is concave down on (a,b) if f' is decreasing on (a,b)

IMP

* a) If $f''(x) > 0$ on (a,b), then f is conca up on (a,b).

b) If $f''(x) < 0$ on (a,b), then f is conca down on (a,b).

* If f is continuous on an open interval containi x_0, & if f changes the direction of its concavity at x_0, then the point $(x_0, f(x_0))$ on th graph of f is called ~~the~~ an INFLECTION POINT f, and we say that f has an inflection poin at x_0.

IMP

RELATIVE EXTREMA; FIRST AND SECOND DERIVATIVE TESTS.

IMP

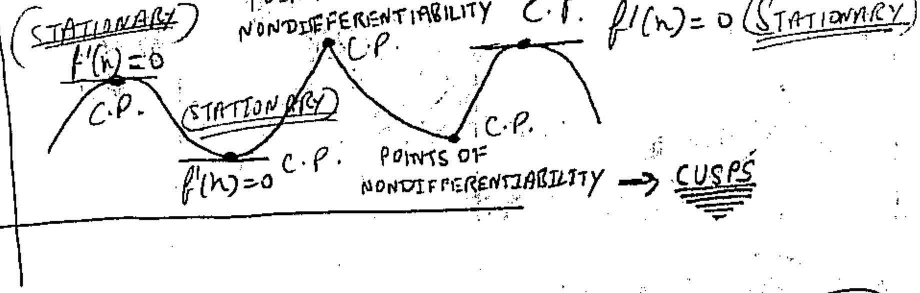
- * A function f is said to have a relative maximum at x_0 if $f(x_0) \geq f(x)$ for all x in some open interval containing x_0.

- * A function f is said to have a relative minimum at x_0 if $f(x_0) \leq f(x)$ for all x in some open interval containing x_0.

- * A function f is said to have a relative extremum at x_0 if it has either a relative max. or min. at x_0.

IMP

- * Relative max or relative min. are not the highest or lowest points of the function.



IMP

- * If f has a relative extremum at x_0 then either $f'(x_0)=0$ or f is not differentiable at x_0.

IMP

- * A CRITICAL POINT for a function f is any value ~~f~~ of x in the domain of f at which $f'(x)=0$ or f is not differentiable; the critical points where $f'(x)=0$ are called STATIONARY POINTS of f.

MOST IMP

- * The relative extrema of a function occur at its critical points though there need not be a relative extremum at every critical point.

↳ 1st derivative

your current paper and electronic subscriptions.

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ACCESS AND NAVIGATION NOTES
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HOW TO DO A DATE RANGE SEARCH IN TECHJOBS

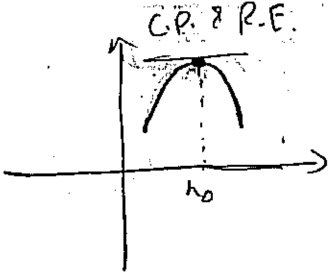
If you wish to look at the jobs listed in a given week on the Techjobs database, follow this format at the search prompt: YYMMDD~YYMMDD. For instance, if you want to see all the jobs posted during the first week of October, type this as your search: 921001~921007.

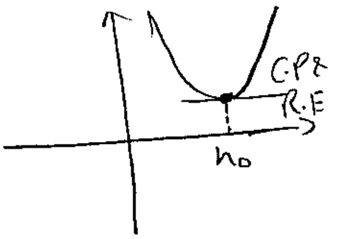
HELP FOR ISN USERS OF LINUS

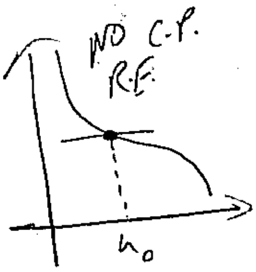
Just after logging into LINUS and before accessing any database, type 'isn' at the prompt. This should help to alleviate the problems users coming in over ISN lines are experiencing.

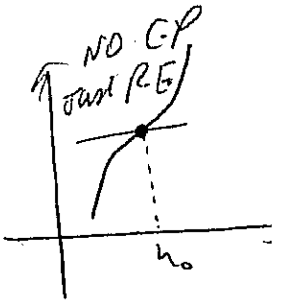
**Editor: Will Rosenthal   mhuxd!wer

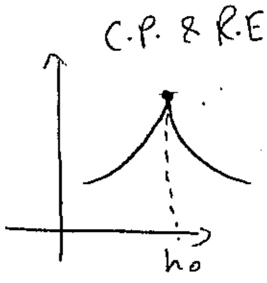
Please send Linus Beat feedback to mhuxd!info

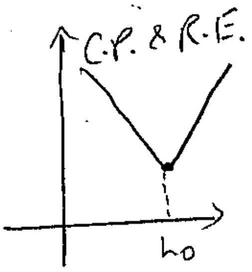


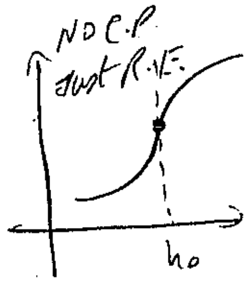


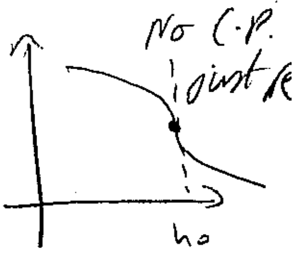












*In Rational functions, C.P. can be found by ~~factoring th~~ ~~NUMERATOR~~. FINDING THE FIRST DERIVATIVE.

- - - - - - - + + + + - - -
* -1 0 1

here C.P = -1, 0, 1
R.E = 0, 1 & not -1

Factoring the numerator gives us the zeros of the function - (Setting f(x) = 0 ~~#~~ in other words

Factoring the denominator gives us the vertical asymptotes or the hole of the function $\frac{P(x)}{Q(x)}$

FIRST DERIVATIVE TEST

* If f is continuous at a critical point x_0

a) If $f'(x)>0$ on an open interval extending left from x_0 and $f'(x)<0$ on an open int extending right from x_0, then f has a relative max. at x_0.

b) If $f'(x)<0$ on an open interval extending left from x_0 and $f'(x)>0$ on an open interval extending right from x_0, then f has a relative min. at x_0

c) If $f'(x)$ has the same sign on an open interval extending left from x_0 or on an open interval extending right from x_0, then f does not have a relative extremum at x_0.

Imp Relative extrema of a continuous function f occurs at those critical points where f changes sign.

2nd DERIVATIVE TEST

MOST IMP

* If f is twice differentiable at stationary pt x

a) If $f''(x_0)>0$, then f has a relative min at

Imp b) If $f''(x_0)<0$ then f has a relative max at

essage-Version: 2
To: sgbali
rom: !march
ate: Wed Oct 07 15:01:10 EDT 1992
A-Content-ID: <PMX-LAN-2.2.1-******-hostar-march-1514>
nd-of-Header:
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A-Message-ID: <winPMXSTAR-2.2.1b-march-XYXYXYXYXY-138>
o: !sgbali
nd-of-Protocol:
ontent-Type: Text
ontent-Length: 595

hri,

hen you and I takled on thurs. about how we would conduct the interviews when e went out to the OSWF, I stated that I thought that the work we had done to ate was to identify the OSs used by the workcenters and OSWF and that what we eeded to do now was to identify the functions that each of the OSs performs. hat is what we will be doing when we visit the OSWF and MMOCs. But we (us and ill Blood) never discussed doing that for the workcenters. Who will do that ork? That work has to get done in order to be in a position to recommend any ptimizations.

arc

GRAPHS OF POLYNOMIALS AND RATIONAL FUNCTIONS

HOW TO GRAPH A POLYNOMIAL $P(x)$

Step 1: Find out if the graph is symmetric to origin or y-axis, find if it crosses the x-axis or the y-axis ~~or the asymptotes~~.

Step 2: Calculate $P'(x)$ & $P''(x)$

IMP $P'(x)=0$

Step 3: From $P'(x)$, determine the stationary pts and the intervals where P is increasing or decreasing. Find out if it has vertical tangents.

Step 4: From $P''(x)$ determine the inflection p and the intervals where P is concave up and concave down.

Step 5: Check for the behavior of graph as P approaches ∞.

$\lim_{x\to\infty} P(x) = \lim_{x\to\infty^+} P$ 8

$\lim_{x\to\infty^-} P$

GRAPHS OF RATIONAL FUNCTIONS

* If $P(x)$ & $Q(x)$ are polynomials, then their ratio $f(x) = \frac{P(x)}{Q(x)}$ is called a rational function of x.

* A line $x = x_0$ is called a vertical asymptote for the graph of a function f if $f(x) \to +\infty$ or $f(x) \to -\infty$ as x approaches x_0 from the right or from the left. A line $y = L$ is called a horizontal asymptote for the graph of a function f if $f(x) \to L$ as $x \to +\infty$ or $x \to -\infty$

Steps to graph a Rational Funct

Step 1: Look for symmetry where the curve crosses x & y-axis & the asymptotes { P = eq for asymptotes }.
Look for Oblique Asymptotes.

Step 2: Find the x-intercepts of $P(x)$. At these values we have $f(x) = 0$, so that the graph intersects the x-axis at those points.

IMP PRACT

Step 3: Find the x-intercepts of $q(x)$. At ~~the~~ these values, $f(x)$ approaches $+\infty$ or $-\infty$ and the graph has a vertical asymptote or a HOLE. IMP

* Factoring the denominator gives vertical Asymptote.

Step 4: Compute $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. If either limit has a finite value L, then the line $y = L$ is a horizontal asymptote. $\frac{x^2}{x^2}$

IMP

eg $\lim_{x\to\infty} \frac{x^2}{x^2} = 1$ ↳ coefficient value, $\lim_{x\to\infty} \frac{x}{x^2} = 0$, $\lim_{x\to\infty} \frac{x^2}{x} =$

Step 5: The only places where $f(x)$ can change sign are at the points where the graph intersects the x-axis or has a vertical asymptote. Calculate a sample value of $f(x)$ in each of the open intervals ~~to~~ determined ~~of~~ by these points to see whether the graph is above or below the x-axis over the interval.

IMP

Step 6: From $f'(x)$ & $f''(x)$ determine the stationary points, inflection points, intervals of INC. & dec. upwards & downwards concavity

* Beware of vertical tangents

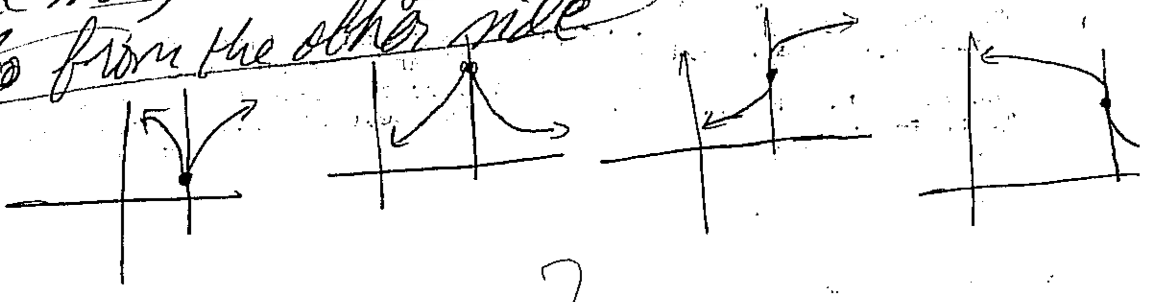
Step 7 - (IMP) Find if the graph crosses the Horizontal Asym by setting f = eq of the Horiz. Asymptote

Step 8: Find the behavior of the graph as x approaches $-\infty$ & $+\infty$. ?

OTHER GRAPHING PROBLEMS

* The graph of a function f is said to have a vertical tangent ~~line~~ at x_0 if f is continuous at x_0 and $|f'(x)|$ approaches $+\infty$ as $x \to x_0$, or if $f'(x)$ is undefined at x_0.

* The graph of a function f is said to have a cusp at x_0 if f is continuous at x_0 and $f'(x) \to +\infty$ as x approaches x_0 from one side, while $f'(x) \to -\infty$ as x approaches x_0 from the other side.



* OBLIQUE ASYMPTODE → ?

If a rational function $P(x)/Q(x)$ is such that the degree of the numerator exceeds the degree of the denominator by one, then the graph of $P(x)/Q(x$ will have an OBLIQUE ASYMPTODE.

$*y = \frac{n^2 - 2}{n}$

IMP

$\boxed{O \cdot A = \lim_{n \to \infty} f(n)}$

$x\text{-int} = \pm 2$

$V \cdot A = n = 0$

$H \cdot A = \text{none}.$

$O \cdot A = \frac{n^2}{n} - \frac{2}{n} \Rightarrow n - \frac{2}{n}$

$\therefore y = n$

$$y' = \frac{n(2n) - (n^2 - 2)}{n^2} = \frac{2n^2 - n^2 + 2}{n^2} = \frac{n^2 + 2}{n^2} \rightarrow \text{cannot sign}$$

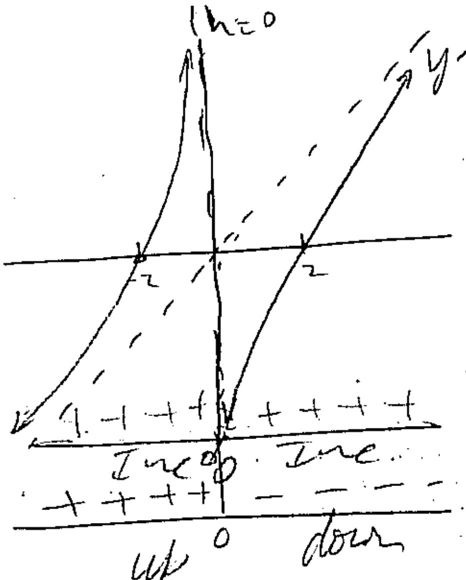
$$\underset{0}{+ + + + + | + + + + +}$$

$$y'' = \frac{n^2(2n) - (n^2+2)(2n)}{n^4} = \frac{2n^3 - 2n^3 - 4n}{n^4}$$

$$= \frac{-4}{n^3}$$

Imp

$$\underset{0}{+ + + + | - - - -}$$



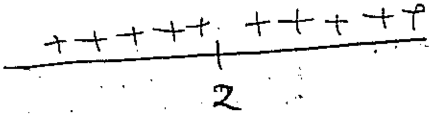
$*y=(x-2)^{1/3}$

$X\text{-int} = (x-2)^{1/3} = 0$

$x-2=0$

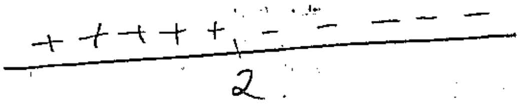
$x=2$

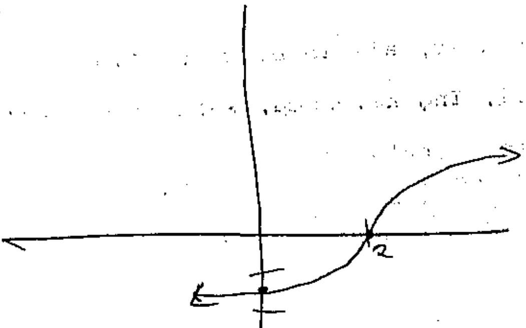
$y' = \frac{1}{3}(x-2)^{\frac{-2}{3}} = \frac{1}{3(x-2)^{2/3}}$



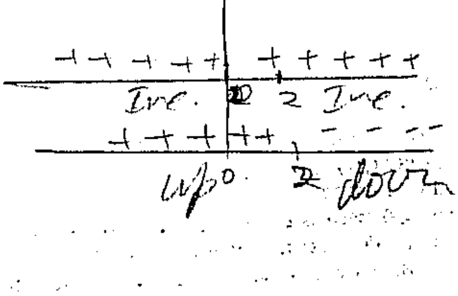
$$y'' = \frac{3(x-2)^{2/3}(0) - 1\left(2(x-2)^{\frac{-1}{3}}\right)}{9(x-2)^{4/3}}$$

$$= \frac{\frac{-2}{(x-2)^{1/3}}}{9(x-2)^{4/3}} = \ominus\frac{2}{9(x-2)^{5/3}}$$





y int = −1.3



G ⇒ Fig.

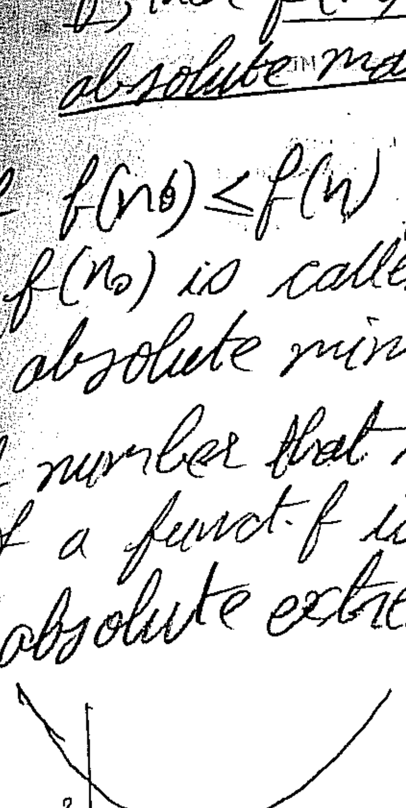
When sign lining a eq. w/ a -ve sign before it you either switch the sign of the final sign line or the sign of any one sign line.

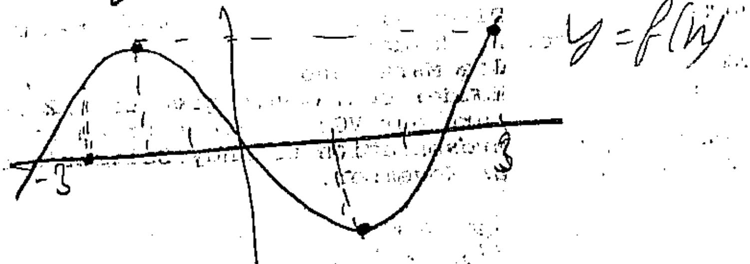
(optimization) MAXIMUM AND MINIMUM VALUES OF A FUNCTION

If $f(x_0) \geq f(x)$ for all x in the domain of the f, then $f(x_0)$ is called the maximum value or absolute maximum value of f.

If $f(x_0) \leq f(x)$ for all x in the domain of f, then $f(x_0)$ is called the minimum value or absolute minimum value of f.

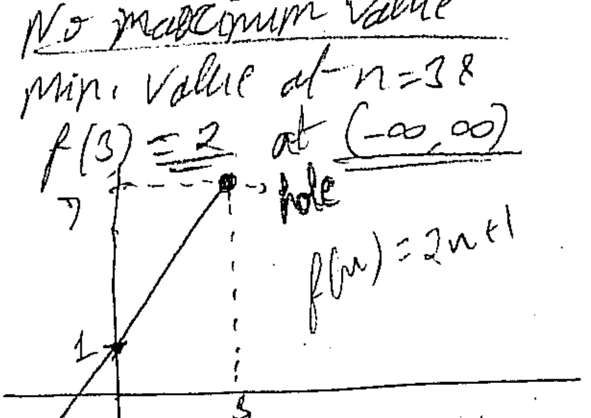
* A number that is either the abs. max or abs. min of a funct. f is called the extreme value or absolute extreme value of f. ↳IS DIFFERENT FROM EXTREMA POINTS } IMP





No maximum value
Min. value at x=3 &
f(3) = 2 at (-∞,∞)

No maximum &
No minimum value.
at (-∞,∞) but has
max & min at [-3,3]



Has a min of 1 & at [0,3)
At [0,3] the max is 7.
There is always a
greater no. when a
hole is present. } IMP

EXTREME VALUE THEOREM - If a function is continuous on a closed interval [a,b] then f has both a maximum & a minimum value ~~at~~ in [a,b]

IMP * THE FUNCTION HAS TO BE CONTINUOUS AND AT A CLOSED INTERVAL.

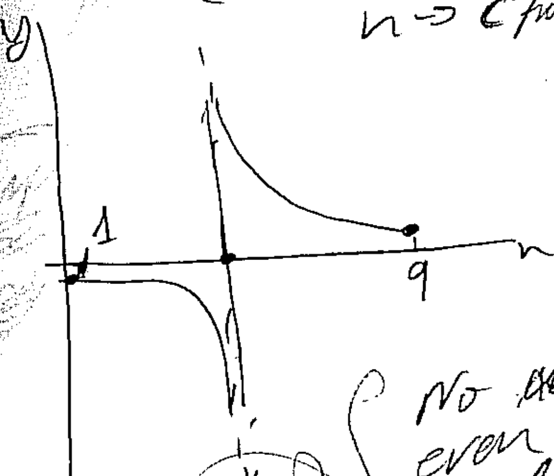
To check for Continuity →

* All polynomials are continuous.
* A function is said to be continuous at [a, if —

(i) It is continuous at (a,b)
(ii) If it is continuoust at a from the right
(iii) If it is continous at b from the left

* A function is continuous at point c if

i) f(c) is defined
ii) $\lim_{x \to c} f(x)$ exists from left + right
(iii) $\lim_{x \to c} f(x) = f(c)$ from left & right



IMP No abs. max. or Abs. min at [1,9] becau even though it's a closed interval & is defined everywhere it has points of discontinuity

* If a function f has an extreme value on a open interval (a,b), then the extreme value occurs at a critical point of f & not on end points ~~at~~ there are no end points in an open interval.

IMP

* EXTREME VALUES OF A FUNCTION f at CLOSED INTERVAL [a,b]

IMP

Step 1: Find the critical points of f in (a,b)

Step 2: Evaluate f at the critical points and the endpoints ~~of~~ a & b

Step 3: The largest of the values in step 2 is t
max & the smallest value is the m

* To find out if a function has extrema points on an ~~open interval~~ of $(-\infty,\infty)$ then we can simply ~~first find the~~ if there are any extrema points & if there are then find ~~the critical points~~ & figure out the extrema points. eg :-

IMP

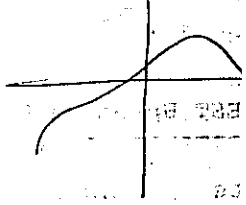
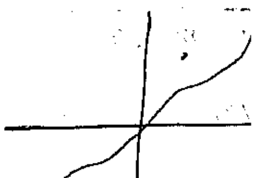
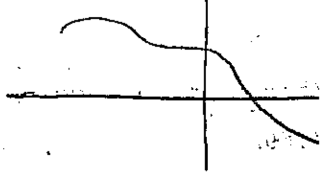
WE SHOULD FIRST FIND THE LIMIT OF THE FUNCTI
APPROACHING INFINITY $(-\infty$ & $+\infty)$

eg :- $f(x) = 3x^4+4x^3$ on $(-\infty,\infty)$

$\lim_{x\to-\infty} 3x^4+4x^3 = -\infty$ $\lim_{x\to\infty^+} 3x^4+4x^3 = \infty$

Has no extrema pts.

$x^{-}(3x+4)$

$\lim_{x\to-\infty} f(x)$	$\lim_{x\to+\infty} f(x)$	Conclusion	Graph
$+\infty$	$+\infty$	Min but no max	
$-\infty$	$-\infty$	max but no min	
$-\infty$	$+\infty$	no max & no min	
$+\infty$	$-\infty$	no max & no min	

* $f(x) = x^4+2x^3-1$ at $(-\infty,+\infty)$

$\lim_{x\to-\infty} x^4+2x^3-1 = +\infty$

$\lim_{x\to+\infty} x^4+2x^3-1 = +\infty$

∴ has a <u>min</u> but no ma

$f'(x) = 4x^3+6x^2$

$2x^2(2x+3)$

$x=0,\ x=\frac{-3}{2}$ } <u>only two C.P.</u>

$f''(x) = 12x^2+12x$

~~12x~~ $12x(x+1)$ - - + +

$x=0,\ x=-1$

$f(0) = -1$ $f(\frac{-3}{2}) = \frac{-43}{16}$

∴ the abs. min is at $\frac{-43}{16}$ when x

* $f(x) = \frac{1}{x^2-x}$

$\lim_{x\to+\infty} \frac{1}{x^2-x} = \lim_{x\to+\infty} \frac{1}{x(x-1)}$

$= +\infty$

$\lim_{x\to-\infty} \frac{1}{x(x-1)} = -\infty$

∴ has max but no min.

$$f'(n) = \frac{n^2-n(0)-1(2n-1)}{(n^2-n)^2}$$

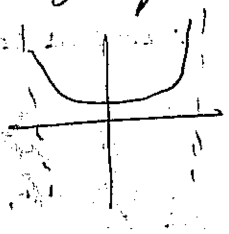
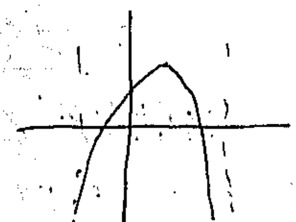
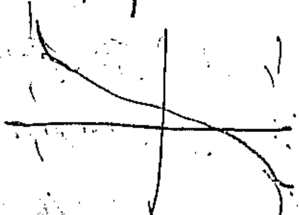
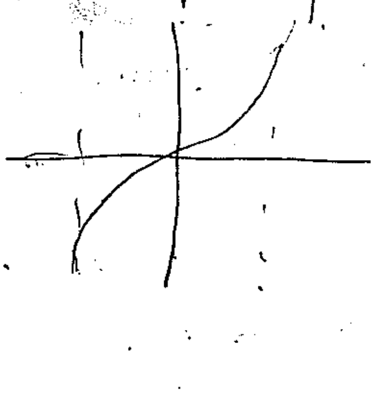
$$= \frac{-2n+1}{(n^2-n)^2} = \frac{-2n+1}{n^4-2n^3+n^2} = \frac{-2n+1}{n^2(n^2-2n+1)}$$

$$= \frac{-2n+1}{n^2(n-1)(n-1)}$$

C.P. =
$n = 1/2$
$n = 0$
$n = 1$

$f(1/2) = -4$
$f(0) =$ undef.
$f(1) =$ undef.

(a, b)

$\lim_{n \to a^+} f(n)$	$\lim_{n \to b^-} f(n)$	Conclusion	Graph
$+\infty$	$+\infty$	min. but no max.	
$-\infty$	$-\infty$	max. but no min.	
$+\infty$	$-\infty$	no max. or min.	
$-\infty$	$+\infty$	No max. or min.	

If any extrema points exist then use the critical points to find them.

Imp

	Hawkins)		
10/21/92	Multiple Business Customers	Michele Lewski	- Sales Effectiveness Rally in Parsippany.
10/22/92	Amdahl	Bali, Bean, Hiering, Das	- Conference 22nd & 23rd, benchmarking visit.
10/28/92	NS, Jan Sharpless	Joe Maranzano	- Mtg on MOSAIC/SDE & NS standard processes.

SPONSORED BY QUEST IN OCTOBER

Date: October 22nd & 23rd
Event: 5th Annual AT&T Quality Conference - Sponsored by QUEST
QUEST Contact: Donna Cullinen, hoqaa!donna, HO 1D-421, (908)949-9204

Purpose: The AT&T Quality Conference will be simulcasted, between the hours of 8AM & 12:15PM, to the HO mini-auditorium on 10/22 & the HO main auditorium on 10/23. Members of QUEST are encouraged to stop by and invite their customers! Some of the conference speakers and their topics are:

Bob Allen - Keynote & "Challenge for '93"
John Imlay, Chairman of D&B Software - "Living the Dream"
Nancy Austin - Author of "A Passion for Excellence"
Joseph M. Juran, Chairman Emeritus of Juran Institute - "World Class Quality: Non-Delegable Roles of Upper Mgmt."

If you have any items for the Customer Events Calendar, send your input to George Grant (HO 1D-406; x6562; hogpa!gsgl; Fax x6868), or directly to homxc!custcal. The next edition will appear on November 4th. Please send in any customer related item(s) for the next release by October 30th.

TOP

* Let f be continuous on an interval I and assume that f has exactly one relative extremum on say at No then -

a) If f has a relative minimum at No, then f(N
is the minimum value of f on the interval I.

b) If f has a relative maximum at No, then f(No).
the maximum value of f on the interval I.

10/

In other words, if f has exactly one relative extremum at interval I then if f has a relative max at No then it is t absolute max & if f has a relative min at No then it is the ~~relative~~ Absolute min.

logical

vertical for oblique asymp. intercepts on ( ) & ( )

MOST IMP

In finding Horiz asymp for Rat. func. we would divide the numerator by the denominator Eg $f(x) = \frac{x^2+3x+1}{x-2}$

$$f(x) = \frac{n(n-2)}{n^2-4} = \frac{n\cancel{(n-2)}}{\cancel{(n-2)}(n+2)} = \frac{n}{n+2}$$

Eg of a HOLE

Hole at n=2

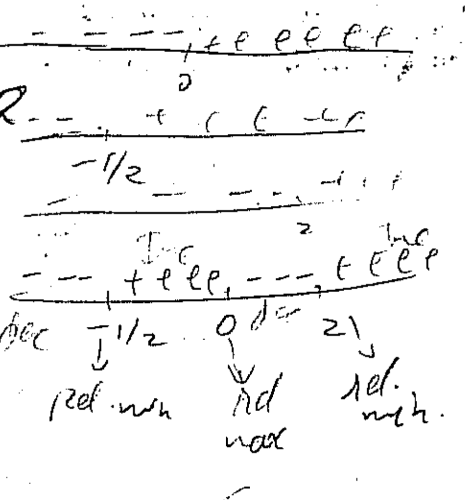
$$y = n^4 - 2n^3 - 2n^2 \quad (-\infty, \infty)$$

$$y' = 4n^3 - 6n^2 - 4n$$

$$2n(2n^2 - 3n - 2)$$

$$2n(2n+1)(n-2)$$

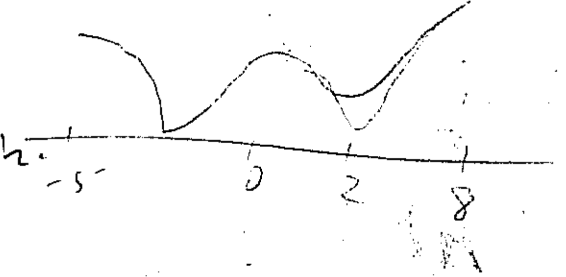
$$n=0,\ n=-1/2,\ n=2$$



$$\lim_{n\to\infty^+} n^4 - 2n^3 - 2n^2 = +\infty$$

$$\lim_{n\to\overline{\infty}} n^4 - 2n^3 - 2n^2 = +\infty$$

∴ there is no abs. max. & we have to find the abs. min.



$f(-1/2) =$

$f(2)$

hostar!SGBALI **Bin: 0**

$f(n) = (1+n)^{2/3}(3-n)^{1/3}$

zeroes = $-1 = n$
$3 = n$

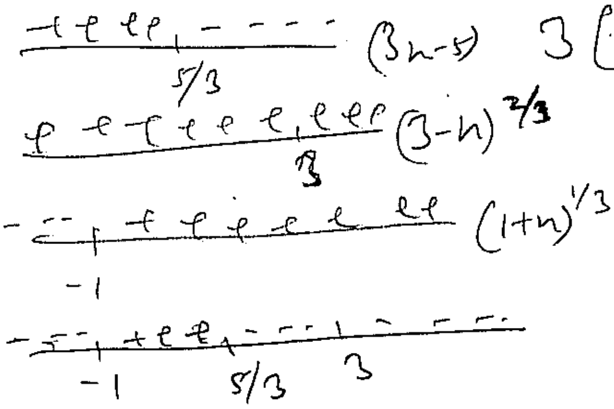
C.P. $f'(n) = (1+n)^{2/3}\frac{1}{3}(3-n)^{-2/3}(-1) + (3-n)^{1/3}\frac{2}{3}(1+n)^{-1/3}$

SGBALI

$$f'(n) = -\frac{1}{3}(3-n)^{-2/3}(1+n)^{2/3} + \frac{2}{3}(1+n)^{-1/3}(3-n)^{+1/3}$$

$$f'(n) = -\frac{1}{3}(1+n)^{-1/3}(3-n)^{-2/3}\left[(1+n) - 2(3-n)\right]$$

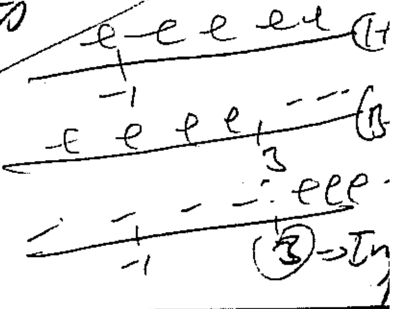
$$= \frac{-1(3n-5)}{3(1+n)^{1/3}(3-n)^{2/3}}$$



C.P $n = 5/3 \to$ Rel.
$n = -1 \Rightarrow$ Rel. mi
$n = 3 \to$ none.

are defined at f(n) & so are not vertical tangents

$$f''(n) = \frac{-32}{9(1+n)^{4/3}(3-n)^{5/3}}$$



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Submitted: 08/27/92 09:49:3
Printed: 08/27/92 09:51:0

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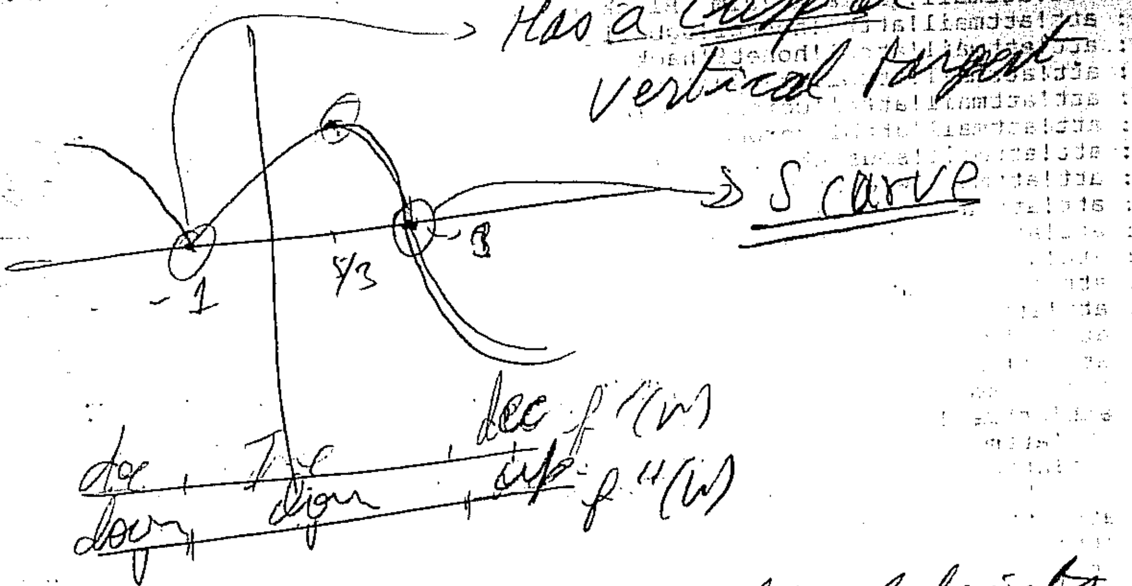
$f(5/3) = 4\sqrt[3]{4/3}$



* Always sign line all the critical points irrespective of the Intervals or a total +ive sign.

* In OPTIMIZATION OR MAX-MIN problems, try to get a formula with NO MORE THAN ONE VARIABLE → MOST IMP

IMP

* When there is a variable underneath the formula, then it is likely to be an open interval problem.

```
rom: att!attmail!finfinis (James G Finfinis )
te: Tue Aug 25 16:15:03 GMT 1992
one: 201-234-3097
bject: Strategic Data Communications Plan - FYI
o: att!attmail!jgottfrid (Joshua N Gottfrid )
o: att!attmail!lschaeffer (Lynne M Schaeffer )
o: att!attmail!heslambolchi (Hossein  Eslambolchi )
o: att!attmail!j2black (Jim  Black )
o: att!attmail!attbl!hostar!sgbali
o: att!attmail!attbl!honet9!hack
o: att!attmail!attbl!granjon!jjbl
o: att!attmail!attbl!corona!tbear
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o: att!attmail!zahmed (Zaheer  Ahmed )
o: att!attmail!dschultz (Dana W Schultz )
o: att!attmail!jlorber (Jennifer C Lorber )
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o: att!attmail!pauthompson (Paulette A. Thompson )
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o: att!attmail!jdharper (James D Harper )
o: att!attmail!kcheung (KwokLeung  Cheung )
o: att!attmail!dbosch (David C Bosch )
o: att!attmail!tingle (John E Tingle )
o: att!attmail!dknichols (David K Nichols )
c: att!attmail!attbl!homxa!jhd
c: att!attmail!ianna (Frank  Ianna )
c: att!attmail!wjcarroll (William J Carroll )
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c: att!attmail!attbl!hugo!las
c: att!attmail!wolfmeyer (Paul A Wolfmeyer )
c: att!attmail!attbl!corona!jam
c: att!attmail!tbrowne (Thomas M Browne )
c: att!attmail!attbl!arch3!paggi
c: att!attmail!joyner (Donald J Joyner )
c: att!attmail!attbl!arch2!pep
c: att!attmail!kuehn (Thomas R Kuehn )
c: att!attmail!dcastro (Daniel R Castro )
c: att!attmail!heyrich (David F Heyrich )
```

CIRCLE = $x^2+y^2=r^2$

or $\pm\sqrt{a-x^2}$

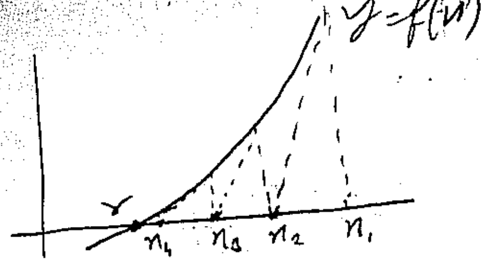
ELLIPSE = $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

HYBERBOLAS = $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

PARABOLAS = $(x-h)^2=\pm 4c(y-k)$

or $(y-k)^2=\pm 4c(x-h)$

NEWTON'S METHOD



IMP

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}} \quad n = 1, 2, 3 \ldots$$

- The First approximation (x_1) can be ~~so~~ made by using the intermediate value theorem. so that it is the smallest no. giving values between ~~so~~ negative & positive)
- Always use Radians (

Roll's Theorem & Mean Value Theorem &
Rectilinear motion. from the Book. &
Pre-Calc. Graphs.

ROLL'S THEOREM =

Let f be differentiable on (a,b) and continuous on [a,b]. If $f(a) = f(b) = 0$, then there is at least one point c in (a,b) where $f'(c) = 0$.

MOST IMP

MEAN VALUE THEOREM

MOST IMP

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$