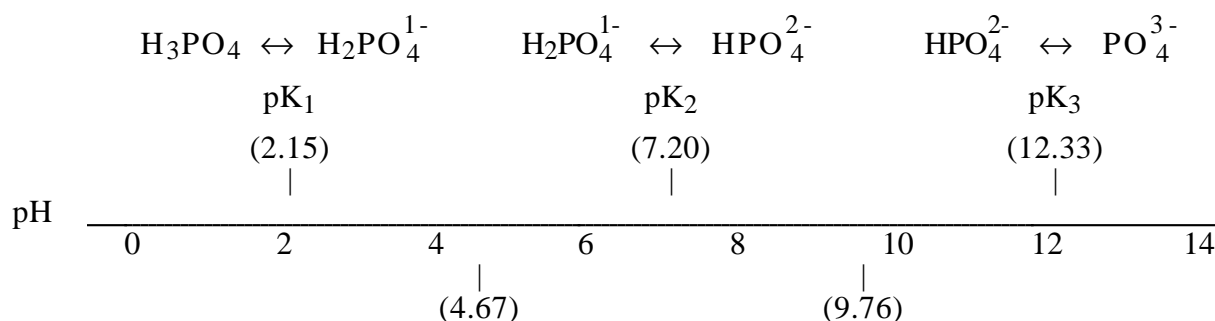# BUFFER PROBLEMS (EXAMPLES WITH SOLUTIONS)

Except in the problems specifically dealing with corrections for activity coefficients, these problems have been solved by using concentrations as if they were activities. The resulting answers are therefore approximate. For many cases, however, the use of activity coefficients is not required for the preparation of biological buffers.

Example 1: Predominant Species of a Buffer Substance as a Function of pH

$$H_3PO_4 \leftrightarrow H_2PO_4^{1-} \qquad H_2PO_4^{1-} \leftrightarrow HPO_4^{2-} \qquad HPO_4^{2-} \leftrightarrow PO_4^{3-}$$

$$pK_1 \qquad\qquad pK_2 \qquad\qquad pK_3$$

$$(2.15) \qquad\qquad (7.20) \qquad\qquad (12.33)$$

pH ___|_____|_____|____

0   2   4   6   8   10   12   14

(4.67)   (9.76)

The relationship shown above indicates the predominant species in solution. Remember that the successive pK_a values of a buffer are related to the successive proton dissociation reactions. Thus, when the pH of the solution has a value equal to a particular pK_a, the two species of the buffer participating in the dissociation described by that pK_a are present in equal concentrations (e.g., $[H_2PO_4^{1-}] = [HPO_4^{2-}]$ at pH 7.20). These are the *predominant species* when the solution pH is near to this particular value. The boundaries of each region are the half-way points between values corresponding to "neighboring" pK's.

1. **What are the predominant species of citrate at pH 5.25?**

Since citric acid is a tricarboxylic acid (see the Table of Common Biological Buffers), it has three pK_a values. These values are 3.13 (pK_1), 4.76 (pK_2), and 6.40 (pK_3). The half-way point between the values for pK_2 and pK_3 is 5.58. Thus, the stated value pH = 5.25 is closer to pK_2 than to pK_3. Accordingly, the predominant species in solution are those represented by the equilibrium of pK_2, namely H_2Cit^{1-} and $HCit^{1-}$.

Examples 2-4:   Calculating the Composition of a Buffer

2.   **What is the composition of 0.1 M sodium acetate buffer, pH 5.00?**

From the Table of Common Biochemical Buffers, acetic acid (HAc, Ac⁻) has a pK_a of 4.76.
Solving the Henderson-Hasselbalch equation

$$pH = pK_a' + \log \frac{c_B}{c_{HB}}.$$

$$5.00 = 4.76 + \log \frac{Ac^{1-}}{HAc}$$

Rearranging gives

$$\log \frac{Ac^{1-}}{HAc} = 5.00 - 4.76 = 0.24, \text{ or}$$

$$\frac{Ac^{1-}}{HAc} = 1.74$$

$$Ac^{1-} = 1.74 \; HAc$$

Substituting into the equation of total buffer concentration ($HAc + Ac^{1-} = 0.1$)

$$HAc + 1.74 \; HAc = 0.1$$

$$HAc = 0.036 \; M$$

$$Ac^{1-} = 1.74 \; 0.036 = = 0.064 \; M$$

From the equation of ionic balance

$$Na^+ = Ac^{1-} = 0.064 \; M$$

So, in the specified buffer, the concentration of acetic acid (HAc) is 0.036 M, while the
concentrations of acetate ion (Ac^{1-}) and sodium ion (Na^+) are 0.064 M.

3.   **Write directions for making 1 L of the buffer described in example 2, using
method A.   What volume of the sodium hydroxide solution will be required?**

> Prepare 500 mL of 0.2 M HAc. Prepare 100 mL of 1.0 M NaOH.
> Dilute the HAc to 900 mL. Add NaOH to pH 4.95. Adjust to final
> volume and pH.

From example 2, 0.064 mol of Ac^{1-} must be produced. This corresponds to the addition of 0.064
mol of NaOH. Since the NaOH solution is 1.0 M, 0.064 L or 64 mL will be required.

4. **What is the composition of 0.05 M potassium phthalate buffer, pH 5.00? Phthalic acid (PH_2, PH^{1-}, P^{2-}) is dibasic with pK_a values of 2.95 and 5.41.**

All three species of the buffer will be present, so two Henderson-Hasselbalch relationships are required to describe the solution equilibria:

$$\text{for } pK_1, \ 5.00 = 2.95 + \log \frac{PH^{1-}}{PH_2};$$

$$\text{for } pK_2, \ 5.00 = 5.41 + \log \frac{P^{2-}}{PH^{1-}} \ .$$

Solving these equations separately gives

$$PH_2 = 9 \times 10^{-3} \ PH^{1-} \qquad \text{or} \qquad PH^{1-} = 112 \ PH_2$$

$$PH^{1-} = 2.57 \ P^{2-} \qquad \text{or} \qquad P^{2-} = 0.390 \ PH^{1-}$$

Substituting into the equation for total buffer concentration ($PH_2 + PH^{1-} + P^{2-} = 0.05$) and solving for PH^{1-} gives

$$9 \times 10^{-3} \ PH^{1-} + PH^{1-} + 0.390 \ PH^{1-} = 0.05$$

$$PH^{1-} = 0.036 \ M$$

Back substituting into the expressions obtained from the individual Henderson-Hasselbalch equations

$$PH_2 = 9 \times 10^{-3} \ PH^{1-} = 3.22 \times 10^{-4} \ M$$

$$P^{2-} = 0.390 \ PH^{1-} = 0.014 \ M$$

As a check to your calculations, the total buffer concentration is

$$PH_2 + PH^{1-} + P^{2-} \ = \ 3.22 \times 10^{-4} + 0.036 \ + 0.014 = 0.05 \ M$$

From the equation for ionic balance, the total K^+ concentration is

$$K^+ = PH^{1-} + 2 \ P^{2-} = 0.064 \ M$$

Examples 5 and 6: Buffer Capacity

5. **How much acid can be generated in a 10 mL reaction mixture, buffered at pH 5.50 with pyridine-chloride, $c_t = 0.1$ M, if the maximum allowable decrease in pH cannot be greater than 0.1 unit?**

Pyridine (P, PH$^+$) is a monoacidic base with $pK_a = 5.23$. To obtain the buffer composition at pH 5.50, solve the Henderson-Hasselbalch equation and the total buffer concentration equations as described above

$$5.50 = 5.23 + \log \frac{P}{PH^+}$$

$$P + PH^+ = 0.1$$

$$P = 0.065 \text{ M} \qquad \text{and} \qquad PH^+ = 0.035 \text{ M}$$

At pH 5.40 (the maximal allowable decrease), the composition is similarly given by

$$5.40 = 5.23 + \log \frac{P}{PH^+}$$

$$P + PH^+ = 0.1$$

$$P = 0.060 \text{ M} \qquad \text{or} \qquad PH^+ = 0.040 \text{ M.}$$

The *increase* in PH$^+$ is is equal to the mmoles of acid generated in the reaction. In 10 mL, this change in pH corresponds a change in concentration of

$$0.040 - 0.035 = 0.005 \text{ M.}$$

$$0.010 \text{ L} \times 0.005 \text{ M} = 5 \times 10^{-5} \text{ moles} = 0.050 \text{ mmoles.}$$

Thus, up to 0.050 mmoles of acid can be formed before the pH change will exceed the set limit.

6. If you use PIPES (PH⁻, P²⁻) at pH 7.00, if you expect 0.25 mmoles of base (OH⁻) to be formed in a 50 mL reaction mixture, and if you want ΔpH to be no greater than 0.05, what must the total buffer concentration be?

The buffer equilibrium at the pH of interest is

$$\text{PIPES} \cdot \text{H}^- \ (\text{PH}^-) \ \leftrightarrow \ \text{PIPES}^- \ (\text{P}^{2-}) \ (\text{p}K_4 = 6.80)$$

Let the subscripts i and f stand for "initial" and "final," respectively. Initially

$$7.00 = 6.80 + \log \frac{P_i^{2-}}{PH_i^-} \ , \qquad \Rightarrow \qquad \frac{P_i^{2-}}{PH_i^-} = 1.58$$

After the reaction is complete

$$7.05 = 6.80 + \log \frac{P_f^{2-}}{PH_f^-} \ , \qquad \Rightarrow \qquad \frac{P_f^{2-}}{PH_f^-} = 1.78$$

The OH⁻ will convert PH⁻ to P²⁻. Since 0.25 mmoles of OH⁻ in 50 mL is 0.005 mmole/mL, the ΔP²⁻ will be 0.005 M. Likewise, the ΔPH¹⁻ will be -0.005 M.

$$P_f^{2-} = P_i^{2-} + 0.005; \qquad\qquad PH_f^{1-} = PH_i^{1-} - 0.005.$$

From the above equations

$$P_i^{2-} = 1.58 \ PH_i^{1-}$$

$$P_i^{2-} + 0.005 = P_f^{2-} = 1.78 \ PH_f^{1-} = 1.58 \ (PH_i^{1-} - 0.005)$$

Solving these two equations simultaneously:

$$PH_i^{1-} = 0.070 \ \text{M}; \qquad\qquad P_i^{2-} = 0.11.$$

Now

$$c_t = PH^- + P^{2-}$$

$$c_t = 0.07 + 0.11 = 0.18$$

The total buffer concentration required is 0.18 M.

Example 7: Buffer Composition With Desired Ionic Strength

7. **Calculate the composition of a glycinamide-chloride (GH^+, G) buffer of pH 8.00, ionic strength 0.05.**

The ionic balance equation, the *equation for ionic strength*, and the Henderson-Hasselbalch equation, respectively, give the following

$$Cl^- = GH^+ = 0.05$$

$$I = \frac{1}{2} \ (GH^+ \times 1 + Cl^- \times 1) = 0.05$$

$$8.00 = 8.20 + \log \frac{G}{GH^+}$$

Substituting the value for GH^+ into the Henderson-Hasselbalch equation and solving for G gives

$$GH^+ = 0.05 \ M \qquad\qquad G = 0.032 \ M$$

$$c_t = GH^+ + G = 0.082 \ M \qquad\qquad Cl^- = 0.05 \ M$$

Example 8: Buffer Composition With Desired Osmolarity

8. **Calculate the composition of a potassium phthalate buffer (PH_2, PH^{1-}, P^{2-}) of pH 5.75 and osmolarity 0.134.**

Although phthalic acid has two pK_a values (4.18, 5.41), only pK_2 is significant at this pH. The amount of free phthalic acid (PH_2) will be negligible. Thus, only one Henderson-Hasselbalch equation is necessary to describe the solution equilibrium

$$5.75 = 5.41 + \log \frac{P^{2-}}{PH^{1-}}$$

For material balance, the equation for osmolarity can be used, while the value for K^+ is obtained from the ionic balance equation

$$M_{os} = PH^{1-} + P^{2-} + K^+ \qquad\qquad K^+ = PH^{1-} + 2 \ P^{2-}.$$

Substituting and solving gives

$$PH^{1-} = 0.016 \ M \qquad\qquad P^{2-} = 0.034 \ M$$

$$c_t = 0.05 \ M \qquad\qquad K^+ = 0.084 \ M$$

Example 9: Calculate the True pH, Given the Thermodynamic pK and Composition by Using an Activity Coefficient Correction

9. **What will be the actual pH, at 25°, of a 0.1 M TRlS-chloride buffer in which the molarity of the free base (TH$^+$) is 0.0333, that of the positive ion (T) 0.0667?**

The Henderson-Hasselbalch equation is

$$pH = 8.06 + \log \frac{T}{\gamma\, TH^+}$$

In order to find γ, first calculate the ionic strength

$$I = \frac{1}{2}\ (TH^+ \times 1 + Cl^- \times 1) = TH^+ = 0.0667$$

$$\sqrt{I} = 0.258$$

Then, calculate the activity coefficient

$$-\log \gamma = \frac{A z^2 \sqrt{I}}{1 + \sqrt{I}} - 0.1\ z^2\ I$$

$$-\log \gamma = \frac{0.512 \times 1 \times 0.258}{1 + 0.258} - 0.1 \times 1^2 \times 0.0667$$

$$= 0.098$$

$$\gamma = 0.798$$

Solving the Henderson Hasselbalch equation with the values for TH$^+$, T, and γ

$$pH = 8.06 + \log \frac{T}{\gamma\, TH^+}$$

$$pH = 8.06 + \log \frac{0.0667}{0.798 \times 0.0333} = 7.86$$

Without using the activity coefficient, the estimated pH value is

$$pH = 8.06 + \log \frac{0.0667}{0.0333} = 7.76$$

Example 10:   True Composition of a Buffer of Given pH and Molarity with Activity Coefficient Correction

10. Calculate the true composition of a 0.05 M potassium phthalate buffer of pH 5.75.

As noted in problem 8, the significant species are PH^- and P^{2-}. The Henderson-Hasselbalch equation, including activity coefficients is

$$5.75 = 5.41 + \log \frac{\gamma_2 \, P^{2-}}{\gamma_1 \, PH^{1-}}$$

$$PH^- + P^{2-} = 0.05$$

Both γ_1 and γ_2 are dependent on the ionic strength, which is dependent on $[\,PH^-]$ and $[P^{2-}]$. Since an analytic solution of this problem is difficult, an iterative approach will be used instead. First, calculate a *first iteration composition*, neglecting activity coefficients

$$P_1 + P_2 = 0.05$$

$$PH^- = 0.016 \text{ M}; \qquad\qquad P^{2-} = 0.034 \text{ M}$$

$$K^+ = P_1 + 2\,P_2 = 0.084 \text{ M}$$

Next, calculate a first iteration ionic strength

$$I = \frac{1}{2} \ (PH^- \times 1 + P^{2-} \times 4 + K^+ \times 1) = 0.119$$

$$I^{1/2} = 0.344$$

Then, calculate first iteration values of γ_1 and γ_2

$$-\log \gamma_1 = \frac{0.512 \times 1 \times 0.344}{1 + 0.344} - 0.1 \times 1 \times 0.119$$

$$-\log \gamma_2 = \frac{0.512 \times 4 \times 0.344}{1 + 0.344} - 0.1 \times 4 \times 0.119$$

$$\gamma_1 = 0.760; \qquad \gamma_2 = 0.333.$$

Now, calculate a *second iteration composition,* using the estimates for the composition and activity coefficients obtained from the first iteration

$$5.75 = 5.41 + \log \frac{0.333 \ P^{2-}}{0.760 \ PH^{1-}}$$

$$P_1 + P_2 = 0.05$$

$$PH^{1-} = 0.00834 \qquad P^{2-} = 0.0417 \qquad K^+ = 0.0917$$

From these values, calculate *second iteration values* for the ionic strength, and the activity coefficients γ_1 and γ_2. Proceeding as above, $\gamma_1 = 0.752$ and $\gamma_2 = 0.320$ are obtained. Now, calculate the composition in a third iteration

$$5.75 = 5.41 + \log \frac{0.320 \ P^{2-}}{0.752 \ PH^{1-}}$$

$$P_1 + P_2 = 0.05$$

$$PH^{1-} = 0.00814 \qquad P^{2-} = 0.0419 \qquad K^+ = 0.0919$$

These third iteration values differ from the second iteration values by only 2 parts in 10,000, and hence these values closely represent the *true composition.* Another round of iteration would not change these values substantially. Note, in this case, that the error in the composition estimated without activity coefficients is substantial. The osmolarity would be 0.142, not 0.134, as assumed in example 8.

Example 11: Change of pH on Dilution

11. Calculate the pH of the buffer described in example 10 after 5-fold dilution to 0.01 M.

In this situation, the composition of the buffer would normally be known, so the calculation is straightforward. The composition after dilution is:

$$PH^{1-} = 0.00163 \qquad P^{2-} = 0.00838 \qquad K^{+} = 0.0184$$

The ionic strength is:

$$I = \frac{1}{2} \ (PH^{1-} \times 1 + P^{2-} \times 4 + K^{+} \times 1) = 0.0268$$

$$\sqrt{I} = 0.164$$

The calculated activity coefficients are

$$\gamma_1 = 0.852 \quad \gamma_2 = 0.527$$

The Henderson-Hasselbalch equation is

$$pH = pK + \log \frac{\gamma_2 \ P^{2-}}{\gamma_1 \ PH^{1-}}$$

$$= 5.41 + \log \frac{0.527 \ P^{2-}}{0.852 \ PH^{1-}}$$

$$= 5.41 + \log \frac{0.527 \times 0.00838}{0.852 \times 0.00163}$$

$$= 5.91$$

The pH has increased by 0.16 unit.